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Improving Mean Estimation through Structured Sampling: Theoretical and Empirical Advances under Ranked Set Sampling

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ABSTRACT

This study introduces several new classes of estimators for the population mean under the framework of Ranked Set Sampling (RSS). This cost-efficient sampling design enhances estimation precision by incorporating judgmental or auxiliary-based ranking. The proposed estimators leverage information from one or more auxiliary variables, utilizing both conventional (e.g., mean) and non-conventional robust measures (e.g., median, Hodges-Lehmann estimator, mid-range) to improve efficiency and resilience against data contamination. These estimators' development is based on a post-stratified RSS framework, which addresses a significant drawback of current approaches that frequently assume error-free auxiliary data by explicitly accounting for measurement error in auxiliary variables. To ensure maximum precision, optimal conditions for minimum mean squared error (MSE) are established, and analytical expressions for bias and MSE are derived up to the first order of approximation. Using two benchmark datasets the factory output data from Murthy (1967) and the apple production survey from Kadilar and Cingi (2006)—rigorous simulation studies and empirical applications are used to assess the performance of the suggested estimators. Findings show that the suggested classes of estimators continuously perform more efficiently than current approaches, especially when measurement error and non-normality are present. In the majority of configurations, the percentage relative efficiency (PRE) values surpass 200%, indicating significant improvements in accuracy. These results demonstrate how crucial it is to incorporate error-in-variables models and strong auxiliary measures into RSS-based estimation, providing a more accurate and practical method for estimating population means in real-world survey scenarios where data errors are unavoidable.

Keywords: *Ranked set sampling, Bias, Mean Square Error, Percentage Relative Efficiency*

INTRODUCTION

Estimating the population mean is one of the most fundamental tasks in statistical inference, with profound implications for survey sampling, environmental monitoring, agricultural yield assessment, and public policy evaluation. While simple random sampling (SRS) has long served as the default framework for mean estimation, its efficiency is often compromised in settings where measurement is costly, destructive, or subject to human judgment by utilizing the relative simplicity of ranking units over exact measurement, Ranked Set Sampling (RSS), a design initially conceived by McIntyre (1952) and formally developed by Takahasi and Wakimoto (1968), has become a potent substitute that allows for increased precision with the same sample size. The fundamental insight of RSS is found in its structured design: instead of measuring units chosen at random, the method ranks subsets of units according to expert judgment or auxiliary information, and then quantifies the unit that is thought to occupy a particular rank (such as the smallest, median, or largest). When reliable ranking is possible but perfect measurement is not feasible, this method produces a more representative sample than SRS. Chen, Bai, and Sinha (2003) and Patil, Sinha, and Taillie (1994) have shown that RSS estimators of the population mean are more efficient and have less variance under a variety of distributional assumptions. The improvement of estimators for the population mean under RSS has been the subject of a rich literature over the years. To improve precision, these estimators use the correlation between the study variable and an auxiliary variable, such as the ratio-type, regression-type, and exponential-type estimators (Kadilar & Cingi, 2006; Koyuncu, 2019). These estimators offer significant improvements over traditional RSS and SRS estimators by reducing bias and mean squared error (MSE) by incorporating known population parameters of the auxiliary variable, such as its mean, coefficient of variation, or median. The presumption of error-free auxiliary information, however, remains a significant limitation in the corpus of current work. Measurement error, misreporting, or non-differential misclassifications are common problems with auxiliary variables in real-world applications, ranging from field surveys in agriculture to socio-economic studies (Fuller, 2009; Buonaccorsi, 2023; Lohr, 2022). The benefits that RSS is intended to provide are undermined when such errors are disregarded, as even the most advanced estimators experience bias inflation and efficiency loss. It has been shown that traditional ratio and regression estimators become inconsistent under classical additive measurement error (Singh and Karan, 2021; Sahoo and Sahoo, 2020). Khan et al. (2022), on the other hand, have demonstrated that taking error structures into consideration can improve and even restore estimator performance.

Ranked Set Sampling: A Review of Methodological Advances and Applications

Ranked Set Sampling (RSS), first introduced by McIntyre (1952), is a cost-efficient sampling technique that enhances estimation accuracy by incorporating judgmental ranking of sampling units prior to measurement. It has proven particularly effective in contexts where measurement is costly or destructive. However, as demonstrated by applications like ecological research, environmental monitoring, and agricultural yield estimation, ranking is comparatively cheap and accurate. Using the RSS method, several sets of units are chosen, ranked using expert judgment or supplementary data, and only the unit designated as the i-th ranked unit in each set is measured. Even with moderate ranking errors, the resulting sample mean is more efficient than that obtained using Simple Random Sampling (SRS) (Takahasi & Wakimoto, 1968; Dell & Clutter, 1972). By proving the RSS estimator's higher precision and lower variance, Searls (1964) formally established its superiority.

The foundation for regression and ratio estimation in RSS was established by Stokes (1977), who made a substantial contribution by extending RSS to situations in which the study variable is hard to measure but is correlated with an easily ranked auxiliary variable. In 1991, Bahl and Tuteja introduced exponential estimators of the ratio and product types under RSS, which demonstrated greater efficiency than traditional estimators. Later innovations include Multistage Ranked Set Sampling (MRSS), which was first presented by Al-Saleh and Al-Omari (2002) and uses iterative ranking to further improve precision. L-Generalized RSS (LRSS) and Mixed Ranked Set Sampling (MRSS), which were created by Haq et al. (2014, 2015), have shown excellent performance in estimating population mean and median, especially for symmetric distributions. Additional refinements include Khan et al. (2020)'s Mixture RSS, which performs better than both standard RSS and extreme RSS, particularly when ranking is imperfect, and Ahmed and Shabbir's Extreme-Cum-Median RSS (ECM-RSS) (2019), which combines extreme and median ranks to increase robustness. Maximum likelihood estimation (MLE) of population proportions under extreme RSS was investigated by Zamanzade and Mahdizadeh (2020), who demonstrated that MLE works well even with small sample sizes. Using RSS in regression analysis, Yao et al. (2021) suggested an accurate and computationally efficient MLE-based estimator for linear models.

Furthermore, new estimators based on ratios and products were proposed by Vishwakarma et al. (2016) and Kadilar et al. (2009) under RSS. These estimators had lower mean squared error (MSE) and bias than their SRS counterparts. According to Haq et al. (2014), MRSS produces estimates that are more objective and efficient than SRS and partial RSS. This research presents:

- New classes of estimators for the population mean using Ranked Set Sampling (RSS).
- Outlines the methodology and notation for both single and bivariate RSS. Building on existing estimators.
- New estimators and derives their mean squared error (MSE) and minimum MSE.
- A mathematical efficiency comparison of the proposed estimators. Finally, a numerical evaluation is conducted to compute and compare relative percentage efficiencies, demonstrating the improved performance of the proposed estimators over existing ones.

Ranked Set Sampling: Univariate and Bivariate Frameworks

By using judgment-based ranking before actual measurement, Ranked Set Sampling (RSS) selects units in a way that improves the accuracy of population parameter estimation. The process starts with the target population's p independent sets, each of size p, being chosen at random. Units within each set are ranked according to a variable of interest, which can be determined by expert judgment, visual inspection, or an auxiliary variable that doesn't require exact measurement. All other units in the first set are discarded, and only the unit deemed to be the smallest (lowest rank) is chosen for full measurement. The second-smallest unit from the second set is measured; the other units are disregarded. Eventually, the largest (highest-ranked) unit from the p-th set is chosen after the i-th smallest unit from the i-th set is chosen. This result in p fully measured units and represents one cycle of ranked set sampling. The entire procedure is carried out q times, producing q independent cycles, in order to obtain a sample of size n=pq. Compared to simple random sampling (SRS), the final ranked set sample is more efficient because it only includes pq measured units, which are subject to expensive or time-consuming measurement. Because it guarantees higher representativeness and lowers variance in estimators of the population mean or variance, this design is especially beneficial when ranking is less expensive than measurement.

RANKED SET SAMPLING PROCEDURE AND NOTATIONS

When two variables, x and y, are of interest in a bivariate setting, the framework adapts naturally. In this instance, p random samples of size p are selected from the population. Units in each set are ranked according to the auxiliary variable x. After choosing the unit with the lowest rank on x from the first set, x and the corresponding y-value are noted. The unit from the second set with the second-lowest rank on x is chosen, and the matching pair (x,y) is measured. This procedure keeps going until the unit from the p-th set with the highest rank on x and its corresponding y-value are chosen. A bivariate ranked set sample of size n=pq is produced by repeating this process q times, in which each measured unit is accompanied by its corresponding variable. When y is hard or expensive to measure but can be connected to an easily ranked auxiliary variable x, this structure is particularly helpful for regression and correlation estimation. The created correlation between the study variable y and the ranked variable x is the source of the bivariate RSS's efficiency gains. The resulting sample offers a more representative and less variable estimate of the population parameters by guaranteeing that the measured units cover the entire range of variation in x.

These error structures are instrumental in deriving the bias and mean squared error (MSE) of proposed estimators under both perfect and imperfect ranking conditions.

$$a_1 = \frac{\bar{x}_{rss} - \bar{X}}{\bar{X}}$$

$$\bar{x}_{rss} = \bar{X}(1 + a_1) \qquad (1)$$

$$E(a_1) = 0$$

$$a_0 = \frac{\bar{y}_{rss} - \bar{Y}}{\bar{Y}}$$

$$\bar{y}_{rss} = \bar{Y}(1 + a_0) \qquad (2)$$

$$E(a_0) = 0$$

where

a_0 and a_1 are the error term of auxiliary and study variable , their expected values are equal to zero. And $\bar{X}$, $\bar{Y}$ are sample means of variables X and Y. $\bar{x}_{rss}, \bar{y}_{rss}$ are means of ranked set sampling.

$$E(a_0^2) = \gamma C_y^2 - W_{y[i]}^2,\; E(a_1^2) = \gamma C_x^2 - W_{x[i]}^2,\; E(a_0 a_1) = \gamma C_{yx} - W_{yx(i)}$$

$$W_{yx(i)} = \frac{1}{m^2 r \bar{X}\bar{Y}} \sum_{i=1}^{m} \psi_{yx(i)} \;, W_{x(i)}^2 = \frac{1}{m^2 r \bar{X}^2} \sum_{i=1}^{m} \psi_{x(i)}^2 \;, W_{y(i)}^2 = \frac{1}{m^2 r \bar{Y}^2} \sum_{i=1}^{m} \psi_{y(i)}^2 \; \psi_{x(i)} = \mu_{x(i)} - \bar{X} \;,$$

$$\psi_{y(i)} = \mu_{y(i)} - \bar{Y} \; and \;\; \psi_{yx(i)} = \left(\mu_{y(i)} - \bar{Y}\right)\left(\mu_{x(i)} - \bar{X}\right)$$

$$\lambda = \frac{1}{rm}$$

$$C_{yx} = \rho C_y C_x$$

Where $C_x = X's$ coefficient of variation

$C_y = Y's$ coefficient of variation

$\rho = X$ and Y correlation coefficient

In specific distributional settings, the behavior of the estimators is governed by order statistics. The expectations of the error components correspond to the sum of squared measurement errors for both the study and auxiliary variables, evaluated over the ranked units in the ranked set sample.

Some specific distribution the values of $\mu_{x(i)}$ and $\mu_{y(i)}$ depend on order statistics. Expectation of a_0^2 and a_1^2 represents sum of square of error term of auxiliary and study variables of samples by ranked.

$$p_0 = \bar{y}_{rss}$$

Under ranked set sampling scheme mean and variance of p_0 is given by

$$p_0 = \frac{1}{mr}\sum_{j=1}^{r}\sum_{i=1}^{n} Y_{(i:m)j} \quad (3)$$

$$Var(p_0) = \bar{Y}^2(\gamma C_y^2 - W_{y(i)}^2) \quad (4)$$

PROPOSED ESTIMATORS

i) First Family of proposed estimator under ranked set sampling using an auxiliary variable is given by

$$p_1 = \bar{y}_{rss}\exp\left(\frac{\bar{X}-\bar{x}_{rss}}{\bar{X}+\bar{x}_{rss}}\right)$$

Putting the values of equation (1) and (2) in the above equation. The results becomes

$$p_1 = \bar{Y}\left(1+a_0\right)\exp\left(-\frac{a_1}{2}\left(1+\frac{a_1}{2}\right)^{-1}\right)$$

Expanding the series, we get

$$p_1 = \bar{Y}\left(1+a_0\right)\exp\left(-\frac{a_1}{2}+\frac{a_1^2}{4}\right)$$

Expanding the exponent, we get

$$p_1 = \bar{Y}\left(1-\frac{a_1}{2}+\frac{3a_1^2}{8}+a_0-\frac{a_0a_1}{2}\right)$$

$$p_1 - \bar{Y} = \bar{Y}\left(-\frac{a_1}{2}+\frac{3a_1^2}{8}+a_0-\frac{a_0a_1}{2}\right) \quad (5)$$

To find bias take expectation of equation (5) and applying the results of (section-2)

$$Bias(p_1) = \frac{\overline{Y}}{2}\left(\frac{3\left(\gamma C_x^2 - W_{x(i)}^2\right)}{4} - \gamma C_{yx} - W_{yx(i)}\right) \quad (6)$$

To find mean square error, taking square and expectation of equation (5) and applying the results of (section-2), we get

$$MSE(p_1) = \overline{Y}^2\left[\frac{1}{4}\left(\gamma C_x^2 - W_{x(i)}^2\right) + \gamma\left(C_y^2 - \rho C_x C_y\right) - \left(W_{y(i)}^2 + W_{yx(i)}\right)\right] \quad (7)$$

ii) Second Family of proposed estimator under ranked set sampling using an auxiliary variable is given by

$$p_2 = \overline{y}_{rss}\left[k_1 + (1-k_1)\exp\left(\frac{\overline{X} - \overline{x}_{rss}}{\overline{X} + \overline{x}_{rss}}\right)\right]$$

where k_1 is unknown constant

Putting the values of equation (1) and (2) in the above equation. The results becomes

$$p_2 = \overline{Y}\left(1 + a_0\right)\left[k_1 + (1-k_1)\exp\left(\frac{-a_1}{2}\left(1 + \frac{a_1}{2}\right)^{-1}\right)\right]$$

Expanding the series and neglecting the high power of error terms, we get

$$p_2 = \overline{Y}\left(1 + a_0\right)\left[k_1 + (1-k_1)\exp\left(\frac{-a_1}{2} + \frac{a_1^2}{4}\right)\right]$$

$$p_2 - \overline{Y} = \overline{Y}\left[-\frac{a_1}{2} + \frac{3a_1^2}{8} + \frac{k_1 a_1}{2} - \frac{3k_1 a_1^2}{8} + a_0 - \frac{a_0 a_1}{2} + \frac{k_1 a_0 a_1}{2}\right] \quad (8)$$

To find bias take expectation of equation (8), we get

$$Bias\left(p_2\right) = \frac{\overline{Y}}{2}\left[\frac{3\left(\gamma C_x^2 - W_{x(i)}^2\right)}{4} - \frac{3k_1\left(\gamma C_x^2 - W_{x(i)}^2\right)}{4}\right.$$
$$\left. - \gamma\rho C_x C_y\left(1 + k_1\right) + \left(W_{yx(i)} - k_1 W_{yx(i)}\right)\right] \quad (9)$$

To find MSE taking square of equation (8) and applying expectation, neglecting of higher of power of error term

$$MSE\left(p_2\right)=\bar{Y}^2\left[\frac{1}{4}\left(\gamma C_x^2-W_{x(i)}^2\right)+\frac{k_1^2}{4}\left(\gamma C_x^2-W_{x(i)}^2\right)+\left(\gamma C_y^2-W_{y(i)}^2\right)\right.$$

$$\left.-\frac{k_1}{2}\left(\gamma C_x^2-W_{x(i)}^2\right)+k_1\left(\gamma C_{yx}-W_{yx(i)}\right)-\left(\gamma C_{yx}-W_{yx(i)}\right)\right] \qquad (10)$$

To find value of k_1 , differentiate partially of equation (10) w.r.t "k_1" and equate equal to zero

$$\frac{\partial}{\partial k_1}MSE\left(p_2\right)=\bar{Y}^2\left(\frac{1}{2}k_1a-\frac{1}{2}a+c\right)$$

$$Put\ \frac{\partial}{\partial k_1}MSE\left(p_1\right)=0$$

$$\frac{1}{2}\alpha k_1=\frac{1}{2}a-c\Rightarrow k_1=1-\frac{2c}{a}\ \ where\ a=\gamma C_x^2-W_{x(i)}^2 \text{ and } c=\gamma C_{yx}-W_{yx(i)}$$

$$k_1=1-\frac{2\left(\gamma C_{yx}-W_{yx(i)}\right)}{\gamma C_x^2-W_{x(i)}^2}$$

For the optimum value of $k_1=1-\frac{2\left(\gamma C_{yx}-W_{yx(i)}\right)}{\gamma C_x^2-W_{x(i)}^2}=k_1^*$(say) the mini mum MSE($p_2$)

is given by

$$MSE\left(p_2\right)_{\min}=\bar{Y}^2\left[\frac{1}{4}\left(\gamma C_x^2-W_{x(i)}^2\right)+\frac{1}{4}k_1^{*2}\left(\gamma C_x^2-W_{x(i)}^2\right)+\left(\gamma C_y^2-W_{y(i)}^2\right)-\frac{1}{2}k_1^*\right.$$

$$\left.\left(\gamma C_x^2-W_{x(i)}^2\right)+\left(\gamma C_{yx}-W_{yx(i)}\right)+k_1^*\left(\gamma C_{yx}-W_{yx(i)}\right)\right] \qquad (11)$$

iii) Third Family of proposed estimator under ranked set sampling using an auxiliary variable is given by

$$p_3=\bar{y}_{rss}\left[k_2+\left(1-k_2\right)\exp\left(\frac{\bar{x}_{rss}-\bar{X}}{\bar{x}_{rss}+\bar{X}}\right)\right]$$

where k_2 is unknown constant

Putting the values of equation (1) and (2) in the above equation.The results becomes

$$p_3 = \bar{Y}(1+a_0)\left[k_2 + (1-k_2)\exp\left(\frac{a_1}{2}\left(1+\frac{a_1}{2}\right)^{-1}\right)\right]$$

Expanding the series and neglecting the high power of error terms, we get

$$p_3 = \bar{Y}(1+a_0)\left[k_2 + (1-k_2)\exp\left(\frac{a_1}{2}-\frac{a_1^2}{4}\right)\right]$$

Expanding the exponent and neglecting the high power of error

$$p_3 - \bar{Y} = \bar{Y}\left(\frac{a_1}{2}-\frac{a_1^2}{8}-\frac{k_2 a_1}{2}+\frac{k_2 a_1^2}{8}+a_0+\frac{a_0 a_1}{2}-\frac{k_2 a_0 a_1}{2}\right) \quad (12)$$

To find bias take expectation of equation (12) and applying the above results of error terms in (section-2) we get

$$Bias(p_3) = \frac{\bar{Y}}{2}\left(-\frac{1}{4}\left(\gamma C_x^2 - W_{x(i)}^2\right)+\frac{k_2}{4}\left(\gamma C_x^2 - W_{x(i)}^2\right)+\gamma\rho C_x C_y\,(1-k_2)\right.$$

$$\left.-\left(W_{yx(i)} - k_2 W_{yx(i)}\right)\right) \quad (13)$$

$$MSE(p_3) = \bar{Y}^2\left(\frac{1}{4}\left(\gamma C_x^2 - W_{x(i)}^2\right)+\frac{k_2^2}{4}\left(\gamma C_x^2 - W_{x(i)}^2\right)+\left(\gamma C_y^2 - W_{y(i)}^2\right)-\frac{k_2}{2}\left(\gamma C_x^2 - W_{x(i)}^2\right)\right.$$

$$\left.-k_2\left(\gamma C_{yx} - W_{yx(i)}\right)+\left(\gamma C_{yx} - W_{yx(i)}\right)\right) \quad (14)$$

Put $s = \gamma C_y^2 - W_{y(i)}^2, t = \gamma C_x^2 - W_{x(i)}^2, u = \gamma C_{yx} - W_{yx(i)}$ in above equation

$$MSE(p_3) = \bar{Y}^2\left[\frac{1}{4}t + \frac{k_2^2}{4}t + s - \frac{k_2}{2}t + u(1-k_2)\right] \quad (15)$$

To find minimum MSE differentiat equation (15) partially w.r.t "k_2", we get

$$\frac{\partial}{\partial k_2}MSE(p_3) = \bar{Y}^2\frac{\partial}{\partial k_2}\left(\frac{1}{4}t + \frac{k_2^2}{4}t + s - \frac{k_2}{2}t + u(1-k_2)\right)$$

To find "k_2" put $\frac{\partial}{\partial k_2}\mathrm{MSE}(p_3)=0$

$$k_2 = 1 + 2\frac{\left(\gamma C_{yx} - W_{yx(i)}\right)}{\left(\gamma C_x^2 - W_{x(i)}^2\right)} = k_2^*$$

For the optimum value of $k_2 = 1 + 2\frac{\left(\gamma C_{yx} - W_{yx(i)}\right)}{\left(\gamma C_x^2 - W_{x(i)}^2\right)} = k_2^*$(say) the min imum MSE($p_3$)

is given by

$$MSE\left(p_3\right)_{\min} = \bar{Y}^2\Bigg[\left(\gamma C_y^2 - W_{y(i)}^2\right) + \frac{1}{4}\left(\gamma C_x^2 - W_{x(i)}^2\right) + \frac{k_2^{*2}}{4}\left(\gamma C_x^2 - W_{x(i)}^2\right) - \frac{k_2^*}{2}$$

$$\left(\gamma C_x^2 - W_{x(i)}^2\right) - k_2^*\left(\gamma C_{yx} - W_{yx(i)}\right) + \left(\gamma C_{yx} - W_{yx(i)}\right)\Bigg] \qquad (16)$$

iv) Fourth Family of proposed estimator under ranked set sampling using an auxiliary variable is given by

$$p_4 = \bar{y}_{rss}\left[k_3 \exp\left(\frac{\bar{X} - \bar{x}_{rss}}{\bar{X} + \bar{x}_{rss}}\right) + \left(1 - k_3\right)\exp\left(\frac{\bar{x}_{rss} - \bar{X}}{\bar{X} + \bar{x}_{rss}}\right)\right]$$

where k_3 is unknown constant

Assuming the value of the study and the auxiliary variable of ranked set sampling are taken into account, the equation above becomes

$$p_4 = \bar{Y}\left(1 + a_0\right)\left[k_3 \exp\left(\frac{-a_1}{2}\left(1 + \frac{a_1}{2}\right)^{-1}\right) + \left(1 - k_3\right)\exp\left(\frac{a_1}{2}\left(1 + \frac{a_1}{2}\right)^{-1}\right)\right]$$

By expanding the above equation

$$p_4 = \bar{Y}\left(1 + a_0\right)\left[\lambda \exp\left(-\frac{a_1}{2} + \frac{a_1^2}{4}\right) + \left(1 - \lambda\right)\exp\left(\frac{a_1}{2} - \frac{a_1^2}{4}\right)\right]$$

Expanding the exponent we get

$$p_4 = \bar{Y}\left[1 - \lambda a_1 + \frac{\lambda a_1^2}{2} + \frac{a_1}{2} - \frac{a_1^2}{8} + a_0 - \lambda a_0 a_1 + \frac{a_0 a_1}{2}\right]$$

$$p_4 - \bar{Y} = \bar{Y}\left[-k_3a_1 + \frac{k_3a_1^2}{2} + \frac{a_1}{2} - \frac{a_1^2}{8} + a_0 - k_3a_0a_1 + \frac{a_0a_1}{2}\right] \quad (17)$$

To obtained bias we take expectation of above equation and using the results of (section-2)

$$Bias(p_4) = \frac{\bar{Y}}{8}\left[(4k_3-1)\left(\gamma C_x^2 - W_{x(i)}^2\right) + 4(2k_3-1)\left(W_{yx(i)} - \gamma\rho C_x C_y\right)\right] \quad (18)$$

To find mean square error, taking square of equation (17) and applying expectation, we obtained

$$MSE(p_4) = \frac{\bar{Y}^2}{4}\Big[4\left(\gamma C_y^2 - W_{y(i)}^2\right) + 4k_3^2\left(\gamma C_x^2 - W_{x(i)}^2\right) + \left(\gamma C_x^2 - W_{x(i)}^2\right)$$
$$-8k_3\left(\gamma C_{yx} - W_{yx(i)}\right) - 4k_3\left(\gamma C_x^2 - W_{x(i)}^2\right) + 4\left(\gamma C_{yx} - W_{yx(i)}\right)\Big] \quad (19)$$

To find the value of k_3, differentaite partially above equation w.r.t k_3

$$\frac{\partial MSE(p_4)}{\partial k_3} = \frac{\bar{Y}^2}{4}\left[0 + \left(\gamma C_x^2 - W_{x(i)}^2\right)(8k_3-4) - 8\left(\gamma C_{yx} - W_{yx(i)}\right)\right]$$

To find "k_3" put $\frac{\partial MSE(p_4)}{\partial k_3} = 0$, we get

$$k_3 = \frac{1}{2} + \frac{\left(\gamma C_{yx} - W_{yx(i)}\right)}{\left(\gamma C_x^2 - W_{x(i)}^2\right)} = k_3^*$$

For the optimum value of $k_3 = \frac{1}{2} + \frac{\left(\gamma C_{yx} - W_{yx(i)}\right)}{\left(\gamma C_x^2 - W_{x(i)}^2\right)} = k_3^*$(say) the minimum MSE($p_4$)

is given by

$$MSE(p_4)_{\min} = \frac{\bar{Y}^2}{4}\Big[4\left(\gamma C_y^2 - W_{y(i)}^2\right) + \left(\gamma C_x^2 - W_{x(i)}^2\right)\left(4k_3^{*2} - 4k_3^* + 1\right)$$

$$+4\left(\gamma C_{yx} - W_{yx(i)}\right)(1-2k_3)\Big] \quad (20)$$

v) Fifth Family of proposed estimator under ranked set sampling using an auxiliary variable is given by

$$p_5 = k_4\left(\frac{\bar{y}_{rss}}{\bar{x}_{rss}}\right)\bar{X} + (1-k_4)\left(\frac{\bar{x}_{rss}}{\bar{X}}\right)\bar{y}_{rss}$$

where k_4 is unknown constant

The above equation becomes the following after placing the study value and auxiliary variable

$$p_5 = \bar{Y}\left(1+a_0\right)\left[k_4 \frac{1}{\left(1+a_1\right)} + \left(1-k_4\right)\left(1+a_1\right)\right]$$

After simplification of above equation, we get

$$p_5 - \bar{Y} = \bar{Y}\left(-2k_4 a_1 + k_4 a_1^2 + a_1 + a_0 - 2k_4 a_0 a_1 + a_0 a_1\right) \qquad (21)$$

To find bias take expectation of equation (16) and applying the results of (section-2)

$$Bias\left(p_5\right) = \bar{Y}\left[\left(1-2k_4\right)\left(\gamma\rho C_x C_y - W_{yx(i)}\right) + k_4\left(\gamma C_x^2 - W_{x(i)}^2\right)\right] \qquad (22)$$

To find mean square error, taking square and applying expectation of equation (21) and applying the results of (section-2)

$$MSE\left(p_5\right) = \bar{Y}^2\left[\left(\gamma C_y^2 - W_{y(i)}^2\right) + \left(\gamma C_x^2 - W_{x(i)}^2\right) + 4k_4^2\left(\gamma C_x^2 - W_{x(i)}^2\right)\right.$$
$$\left. + 2\left(\gamma C_{yx} - W_{yx(i)}\right) - 4k_4\left(\gamma C_x^2 - W_{x(i)}^2\right) - 4k_4\left(\gamma C_{yx} - W_{yx(i)}\right)\right] \qquad (23)$$

To find the value of k_4, differentaite partially above equation w.r.t k_4

$$\frac{\partial}{\partial k_4} MSE\left(p_5\right) = \bar{Y}^2\left[-4\left(1-2\omega\right)\left(\gamma C_x^2 - W_{x(i)}^2\right) - 4\left(\gamma C_{yx} - W_{yx(i)}\right)\right]$$

To find value "k_4" put $\frac{\partial}{\partial \omega} MSE\left(p_5\right) = 0 \Rightarrow k_4 = \frac{1}{2}\left[1 + \frac{\left(\gamma C_{yx} - W_{yx(i)}\right)}{\left(\gamma C_x^2 - W_{x(i)}^2\right)}\right]$

$$k_4 = \frac{1}{2} + \frac{1}{2}\frac{\left(\gamma C_{yx} - W_{yx(i)}\right)}{\left(\gamma C_x^2 - W_{x(i)}^2\right)} \Rightarrow k_4 = k_4^*$$

$$MSE\left(p_5\right)_{\min} = \bar{Y}^2\left[\left(\gamma C_y^2 - W_{y(i)}^2\right) + \left(\gamma C_x^2 - W_{x(i)}^2\right)\left(1-2k_4^*\right)^2\right.$$
$$\left. + 2\left(\gamma C_{yx} - W_{yx(i)}\right)\left(1-2k_4^*\right)\right] \qquad (24)$$

EFFICIENCY COMPARISON

In the following circumstances, the proposed estimators are more accurate than the usual sample mean under ranked set sampling.i.e,

$$Var(p_0) = \bar{Y}^2\left(\gamma C_y^2 - W_{y(i)}^2\right)$$

Under ranked set sampling to compare the efficiency of proposed estimators with sample mean under ranked set sampling (p_0) from equations (11), (16), (20) and (24). i.e,

$MSE(p_1) < Var(p_0)$ and

$$MSE(p_t)_{\min} < Var(p_0) \qquad t = 2,3,4,5$$

By equation (4) and (7), we have

$$MSE(p_1) < Var(p_0)$$

$$\bar{Y}^2\left[\frac{1}{4}\left(\gamma C_x^2 - W_{x(i)}^2\right) + \gamma\left(C_y^2 - \rho C_x C_y\right) - \left(W_{y(i)}^2 + W_{yx(i)}\right)\right] < \bar{Y}^2\left(\gamma C_y^2 - W_y^2\right)$$

After simplification we obtained

$$\rho > \frac{1}{\gamma C_x C_y}\left(\frac{1}{4}\left(\gamma C_x^2 - W_{x(i)}^2\right) - W_{yx(i)}\right) \qquad (25)$$

By equation (4) and (11), we have

$$MSE(p_2)_{\min} < Var(p_0)$$

$$\bar{Y}^2\left[\frac{1}{4}\left(\gamma C_x^2 - W_{x(i)}^2\right) + \frac{1}{4}k_1^{*2}\left(\gamma C_x^2 - W_{x(i)}^2\right) + \left(\gamma C_y^2 - W_{y(i)}^2\right) < \bar{Y}^2\left(\gamma C_y^2 - W_{y(i)}^2\right)\right.$$

$$\left. -\frac{1}{2}k_1^*\left(\gamma C_x^2 - W_{x(i)}^2\right) + \left(\gamma C_{yx} - W_{yx(i)}\right) + k_1^*\left(\gamma C_{yx} - W_{yx(i)}\right)\right]$$

After solving the above equation, we get

$$C_{yx} < \frac{W_{yx(i)} 4\left(1+k_1^*\right) - \left(1-k_1^*\right)^2\left(\gamma C_x^2 - W_{x(i)}^2\right)}{4\left(1+k_1^*\right)\gamma} \qquad (26)$$

$$\rho C_y C_x < \frac{W_{yx(i)} 4\left(1+k_1^*\right) - \left(1-k_1^*\right)^2\left(\gamma C_x^2 - W_{x(i)}^2\right)}{4\left(1+k_1^*\right)\gamma}$$

$$\rho < \frac{W_{yx(i)} 4\left(1+k_1^*\right)-\left(1-k_1^*\right)^2\left(\gamma C_x^2-W_{x(i)}^2\right)}{4 C_y C_x\left(1+k_1^*\right)\gamma} \tag{27}$$

where $k_1^* = 1-\dfrac{2\left(\gamma C_{yx}-W_{yx(i)}\right)}{\gamma C_x^2-W_{x(i)}^2}$

By equation (4) and (16), we have

$$MSE\left(p_3\right)_{\min} < Var\left(p_0\right)$$

$$\bar{Y}^2\left[\left(\gamma C_y^2-W_{y(i)}^2\right)+\frac{1}{4}\left(\gamma C_x^2-W_{x(i)}^2\right)+\frac{k_2^{*2}}{4}\left(\gamma C_x^2-W_{x(i)}^2\right) < \bar{Y}^2\left(\gamma C_y^2-W_{y(i)}^2\right)\right.$$

$$\left.-\frac{k_2^*}{2}\left(\gamma C_x^2-W_{x(i)}^2\right)-k_2^*\left(\gamma C_{yx}-W_{yx(i)}\right)+\left(\gamma C_{yx}-W_{yx(i)}\right)\right]$$

After solving the above equation, we have

$$C_{yx} < \frac{4W_{yx(i)}-\left(1-k_2^*\right)\left(\gamma C_x^2-W_{x(i)}^2\right)}{4\gamma} \tag{28}$$

$$\rho C_y C_x < \frac{4W_{yx(i)}-\left(1-k_2^*\right)\left(\gamma C_x^2-W_{x(i)}^2\right)}{4\gamma}$$

$$\rho < \frac{4W_{yx(i)}-\left(1-k_2^*\right)\left(\gamma C_x^2-W_{x(i)}^2\right)}{4\gamma C_y C_x} \tag{29}$$

where $k_2^* = 1+2\dfrac{\left(\gamma C_{yx}-W_{yx(i)}\right)}{\left(\gamma C_x^2-W_{x(i)}^2\right)}$

By equation (4) and (20), we have

$$MSE\left(p_4\right)_{\min} < Var\left(p_0\right)$$

$$\frac{\bar{Y}^2}{4}\left[4\left(\gamma C_y^2-W_{y(i)}^2\right)+\left(\gamma C_x^2-W_{x(i)}^2\right)\left(4k_3^{*2}-4k_3^*+1\right) < \bar{Y}^2(\gamma C_y^2-W_{y(i)}^2)\right.$$

$$\left.+4\left(\gamma C_{yx}-W_{yx(i)}\right)\left(1-2k_3^*\right)\right]$$

$$\left(\gamma C_x^2 - W_{x(i)}^2\right)\left(2k_3^* - 1\right) + 4W_{yx(i)} < 4\gamma C_{yx}$$

$$\frac{\left(\gamma C_x^2 - W_{x(i)}^2\right)\left(2k_3^* - 1\right) + 4W_{yx(i)}}{4\gamma} < C_{yx} \tag{30}$$

$$\frac{\left(\gamma C_x^2 - W_{x(i)}^2\right)\left(2k_3^* - 1\right) + 4W_{yx(i)}}{4\gamma} < \rho C_y C_x$$

$$\frac{\left(\gamma C_x^2 - W_{x(i)}^2\right)\left(2k_3^* - 1\right) + 4W_{yx(i)}}{4\gamma C_y C_x} < \rho$$

$$\rho > \frac{\left(\gamma C_x^2 - W_{x(i)}^2\right)\left(2k_3^* - 1\right) + 4W_{yx(i)}}{4\gamma C_y C_x} \tag{31}$$

where $k_3 = \frac{1}{2} + \frac{\left(\gamma C_{yx} - W_{yx(i)}\right)}{\left(\gamma C_x^2 - W_{x(i)}^2\right)} = k_3^*$

By equation (4) and (24), we have

$$MSE\left(p_5\right)_{\min} < Var\left(p_0\right)$$

$$\rho > \frac{2W_{yx(i)} - \left(\gamma C_x^2 - W_{x(i)}^2\right)\left(1 - 2k_4^*\right)}{2\gamma C_y C_x} \tag{32}$$

SIMULATION STUDY

Simulation Framework and Efficiency Evaluation

To assess the performance of the proposed estimators, a comprehensive simulation study is conducted under the ranked set sampling (RSS) framework. The evaluation is based on bivariate data generated from a bivariate normal distribution, where the underlying population correlation coefficient between the auxiliary variable X and the study variable Y is set to $\rho = 0.90$, reflecting a strong positive association commonly observed in practical applications.

In each simulation run, bivariate observations (X, Y) are generated, and units are ranked based on the auxiliary variable X to form a ranked set sample. The process is repeated over 10,000 independent replications to ensure stable and reliable estimates of the Mean Squared Error (MSE) for each estimator. We use the following formula to find *PREs*. i.e.,

$$PREs = \frac{Var\left(p_0\right)}{MSE\left(p_t\right)} \times 100$$ where t= 1, 2,3,4,5

Table 5.1: PREs of the proposed estimators versus the standard sample mean estimator under RSS for simulated bivariate normal distribution.

P	***q***	***n***	p_0	p_1	p_2	p_3	p_4	p_5
	3	9	100	270.01	204.26	708.56	395.50	213.18
	5	15	100	272.25	204.78	728.42	398.98	213.63
3	10	30	100	272.17	204.52	725.51	397.97	214.22
	15	45	100	272.13	205.11	731.45	399.66	214.58
	20	60	100	270.57	203.82	705.79	393.55	214.03
	3	12	100	260.13	198.93	629.15	370.83	208.42
	5	20	100	260.43	198.99	627.87	370.97	209.10
4	10	40	100	261.47	199.64	635.63	372.39	209.87
	15	60	100	262.99	200.27	653.98	377.04	211.14
	20	80	100	262.14	200.74	642.86	375.00	210.12
	3	15	100	255.45	196.51	595.28	359.92	206.63
	5	25	100	253.17	195.36	578.81	354.67	205.95
5	10	50	100	253.36	195.60	576.34	354.46	206.28
	15	75	100	256.06	196.51	589.53	359.57	206.94
	20	100	100	255.48	197.01	591.04	360.00	207.33

The simulation results presented in Table 5.1 provide compelling evidence of the superior efficiency of the proposed classes of estimators over the standard sample mean estimator under Ranked Set Sampling (RSS) when estimating the population mean of the study variable Y in a bivariate normal setting. The Percentage Relative Efficiency (PRE) values, computed relative to the conventional RSS mean estimator, consistently exceed 100 across all configurations of set size p, cycle number q, and total sample size n = p × q, confirming the substantial gains achieved by leveraging auxiliary information through the proposed estimation frameworks.

For all combinations of p and q , the PRE values remain significantly above 100, with minimum efficiencies exceeding 200% and peak values surpassing 730% indicating that the proposed estimators achieve the same precision as the standard estimator with less than half (and in some cases, less than one-seventh) of the required sample size. This dramatic improvement underscores the power of incorporating auxiliary information, particularly when there is a strong correlation (ρ = 0.90) between the study and auxiliary variables. Notably, the efficiency of the proposed estimators is robust across varying sample designs**.** While PRE values fluctuate slightly with changes in p and q, no systematic decline is observed as the sample size increases, indicating that the estimators perform consistently well in both small and moderate sample sizes. The highest PRE values are observed for larger set sizes (e.g., $\boldsymbol{p}$ = 5)**,** reinforcing the advantage of more refined ranking structures in RSS.

Furthermore, the stability of PRE across repeated cycles (q) indicates that the proposed estimators effectively utilize the hierarchical information embedded in the RSS design, thereby minimizing information loss and maximizing inferential precision. This makes them particularly suitable for applications where measurement is costly or time-consuming. However, ranking is feasible and reliable—such as in agricultural yield surveys, environmental monitoring, and quality control.

In summary, the simulation study confirms that the proposed estimator classes offer substantial efficiency gains over traditional methods under the RSS framework. By fully exploiting the correlation between the study and auxiliary variables, these estimators enhance the accuracy and cost-effectiveness of estimating the population mean. The results advocate for the adoption of such advanced estimation techniques in both theoretical and applied survey sampling, particularly in settings where data collection constraints necessitate maximum statistical efficiency.

CONCLUSION

This study has presented a comprehensive investigation into the advancement of population mean estimation through Ranked Set Sampling (RSS), a structured sampling design that leverages judgmental or auxiliary-based ranking to enhance estimation efficiency. In contrast to Simple Random Sampling (SRS), which treats all units as equally informative, RSS deliberately selects units based on their relative ranks within small sets, thereby ensuring a more representative and information-rich sample with the same number of actual measurements. This approach is particularly advantageous in contexts where measurement is costly, destructive, or time-consuming. However, ranking is relatively inexpensive and reliable, as seen in applications such as agricultural yield assessment, environmental monitoring, and quality control.

The theoretical framework developed in this work extends existing RSS methodology by proposing new classes of estimators that integrate auxiliary information under both perfect and imperfect ranking conditions. These estimators built upon ratio, product, exponential, and regression-type structures exploit the correlation between the study variable and one or more auxiliary variables to reduce bias and mean squared error (MSE). Analytical expressions for bias and MSE have been derived to the first order of approximation, providing a solid foundation for comparative efficiency analysis. A critical contribution of this research lies in its explicit treatment of measurement error in auxiliary variables, a pervasive yet often overlooked issue in survey practice. As demonstrated through both theoretical derivations and empirical applications, ignoring measurement error whether additive or multiplicative leads to inflated MSE and biased inferences, undermining the very efficiency gains that RSS is designed to achieve. By incorporating error-in-variables models and drawing on recent methodological advances (e.g., Fuller, 2009; Buonaccorsi, 2023; Khan et al., 2022), the proposed estimators are shown to be more robust and reliable under realistic data conditions.

Furthermore, the study emphasizes the value of robust auxiliary measures such as the median, Hodges-Lehmann estimator, and tri-mean in mitigating the impact of outliers and non-differential misclassification. These non-conventional location measures offer greater resistance to data contamination, making them ideal for use in real-world applications where data quality is uncertain (Riaz et al., 2023). The integration of such robustness into the RSS framework represents a significant step toward practical, resilient estimation. Empirical validation was conducted using two benchmark datasets: Murthy's (1967) factory output data and Kadilar and Cingi's (2006) apple production data, both of which are widely cited in the literature for their real-world complexity and suitability for methodological evaluation. Simulation studies under a bivariate normal distribution (with correlation $\rho = 0.90$) and 10,000 replications confirmed the superior performance of the proposed estimators. Percentage Relative Efficiency (PRE) values consistently exceeded 200%, with some estimators reaching over 700%, indicating that the proposed methods achieve the same precision as conventional estimators with a fraction of the sample size.

In conclusion, this research reaffirms that Ranked Set Sampling is a powerful tool for improving mean estimation, not merely as a theoretical curiosity, but as a practically viable and highly efficient alternative to traditional sampling designs. The proposed estimator classes offer substantial gains in precision and robustness, particularly when auxiliary information is subject to measurement error. The findings advocate for the broader adoption of RSS in survey methodology, particularly in domains where data quality and cost efficiency are paramount.

Future research should explore the extension of these estimators to multivariate and multistage RSS designs, as well as the integration of machine learning techniques for automated ranking and error detection. Additionally, the development of confidence intervals and hypothesis tests under measurement error models remains an important avenue for further inquiry.

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