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A Study of Shewhart Control Charts Using Robust Measures

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ABSTRACT

Statistical Quality Control (SQC) is a structured methodology used to monitor, evaluate, and control industrial production processes to maintain consistent product quality and operational efficiency. Control charts are essential tools widely utilized in business to maintain process variability within acceptable limits. The CUSUM chart is the most effective standard type of control chart, serving as a memorial chart. This study proposes a novel configuration for CUSUM Charts based on the utilization of auxiliary information through a limited number of estimators. It is a collaborative effort to implement traditional location measures to enhance ratio estimators using auxiliary variable information. We have proposed a set of ratio estimators for finite population mean utilizing information from auxiliary variables through both standard and unconventional measures of central tendency. We have amalgamated the tri-mean, Hodges-Lehmann estimator, mid-range, and deciles mean of the auxiliary variables to facilitate the objective. The attributes associated with the proposed set of ratio estimators are evaluated using mean square error. Moreover, resilience to extreme observations (outliers) is an additional attribute of the proposed estimators.

Keywords: *Auxiliary information; performance measures; control charts; Ratio Estimators; Conventional measures; Non-Conventional (robust) measures.*

INTRODUCTION

Statistical Process Control (SPC) plays a pivotal role in maintaining product quality and operational efficiency across manufacturing and service industries. Among the foundational tools of SPC, the Shewhart control chart**,** introduced by Walter A. Shewhart in the 1920s, remains one of the most widely used techniques for monitoring process stability and detecting out-of-control conditions (Montgomery, 2020). The classical Shewhart chart operates by plotting sample statistics typically the sample mean and standard deviation over time, with control limits set at ±3 standard errors from the process mean. These limits are designed to distinguish between common-cause variation (inherent to the process) and special-cause variation (indicative of assignable disruptions).

Despite its historical significance and widespread adoption, the conventional Shewhart chart suffers from a critical limitation: it relies heavily on the assumptions of normality, homogeneity, and absence of outliers**.** In real-world industrial environments, these assumptions are frequently violated due to measurement errors, instrument malfunctions, human error, or transient process disturbances. Under such conditions, the use of classical estimators (mean and variance) can lead to inflated control limits**,** increased false alarm rates**,** and delayed detection of actual process shifts (Riaz et al., 2023; Abbasi & Miller, 2013).

To address these challenges, researchers have increasingly turned to robust statistical methods that are resistant to contamination and non-normality. Robust estimators minimize the influence of extreme observations while maintaining high efficiency under ideal conditions. In recent years, robust alternatives such as the median, tri-mean, Hodges-Lehmann estimator, mid-range, and quartile deviation (QD) have been proposed as replacements for classical location and scale measures in control charting applications (Riaz & Abbas, 2021; Khan et al., 2022). For instance, Riaz et al. (2023) demonstrated that control charts based on robust measures exhibit superior performance in terms of Average Run Length (ARL) and Relative ARL (RARL) under contaminated data environments. Similarly, Abid et al. (2021) developed robust EWMA and CUSUM charts using trimmed means and QD, showing enhanced sensitivity to small and moderate shifts in the presence of outliers.

Building on this growing body of research, this study proposes a new class of Shewhart-type control charts that utilize robust estimators of location and dispersion derived from auxiliary information. Specifically, we incorporate non-conventional measures such as the tri-mean, mid-range, and quartile deviation to improve the accuracy and reliability of process monitoring. By leveraging auxiliary variables correlated with the study variable, our approach enhances estimator efficiency and reduces sensitivity to measurement error, a common issue in industrial data collection (Singh & Karan, 2021; Fuller, 2009).

Some Important Notations

Many researchers have discussed the tricks for the structure of effective estimators in favor of the population mean. Assume $Z=\{Z_1,Z_2,Z_3,.....,Z_N\}$ that several population units and the sample size *n* is taken from this population as a simple random sample without replacement (*SRSWOR*), provided *n*<*N*. Study *n* pair of observations $(y_i,x_1), i=1,2,3......n$ for the study and supporting variable, respectively. To find the bias and mean squared error (*MSE*) of the estimators, we study the following results

$$e_0=\frac{\bar{y}-\bar{Y}}{\bar{Y}} \text{ and } e_1=\frac{\bar{x}-\bar{X}}{\bar{X}}$$

such that

$$E(e_0)=0\,,E(e_1)=0\,,E(e_0^2)=\phi C_y^2\,,E(e_1^2)=\phi C_x^2 \text{ and } E(e_0e_1)=\phi\rho_{yx}C_yC_x$$

Where

$$\phi=\frac{1}{n}-\frac{1}{N}\,,C_y^2=(\bar{Y}^2)^{-1}S_y^2\,,C_x^2=(\bar{X}^2)^{-1}S_x^2\; S_y^2=(N-1)^{-1}\sum_{i=1}^{N}(y_i-\bar{Y})^2\,,S_x^2=(N-1)^{-1}\sum_{i=1}^{N}(x_i-\bar{X})^2\,,$$

$$S_{yx}^2=(N-1)^{-1}\sum_{i=1}^{N}(y_i-\bar{Y})(x_i-\bar{X})\;\;,\rho_{yx}=(S_yS_x)^{-1}S_{yx}$$

Where S_y^2 and $S_x{}^2$ are the variances of y and x respectively.

Well-known Estimators

A sample survey frequently incorporates supplementary demographic data in contemporary practice. We obtain supplementary information from several sources, including regression, ratios, and products. To enhance the efficiency of the SRSWOR sample, the average ratio and regression estimators are frequently employed when the correlation between the study and auxiliary variables is positively associated under specific conditions. Furthermore, the information addresses various estimators available and the category of estimators derived from average populations. In a sample study, supplementary variable information is typically employed to enhance the accuracy of calculating the average population and the overall value of the survey variables. In specific research, the correlation between population and auxiliary variables may be significant. This introduction presents successful estimates of auxiliary variables for determining the average characteristics of a quality sample, articulated in terms of Bias and MSE when a replacement sample is utilized or the study of Shewhart control charts, we use estimators of Irfan et al. (2018). Some selected estimators of Irfan et al. (2018) are given below:

The simple population mean of simple random sampling is given as:

$$\bar{Y}=\frac{\sum_{i=1}^{n}y}{n}=M_1(say) \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

In case of simple random sampling without replacement (*SRSWOR*), the sample mean $\bar{y}_{SRS}$ is used to estimate population mean Y, which is an unbiased estimator.

The *MSE* of simple random sampling is given below:

$$MSE(\hat{\bar{Y}})=V(\hat{\bar{Y}})=\phi\bar{Y}^2C_y^2 \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Cochran (1940) has presented the ratio estimator for estimating the population mean $\bar{Y}$ of the study variable Y as:

$$\hat{\bar{Y}}_R=\bar{y}\left(\frac{\bar{X}}{\bar{x}}\right),\bar{x}\neq 0\;=\;M_2(say) \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

The bias and *MSE* of the ratio estimator to the first-degree of approximation are given below:

$$B(\hat{\bar{Y}}_R) \cong \phi\bar{Y}(C_x^2 - \rho_{yx}C_xC_y)$$

and

$$MSE(\hat{\bar{Y}}_R) \cong \phi\bar{Y}^2[C_y^2 + C_x^2 - 2\rho_{yx}C_yC_x] \quad \text{..(4)}$$

Similar to ratio estimation, linear regression estimation should use secondary variables that are associated with the dependent variable to increase accuracy. When considering the association between, we can understand that the correlation is almost linear, but the line does not cross the origin. This is an estimate based on linear regression from to, and not on the ratio of two variables. Watson (1937) used a linear unbiased regression estimator as:

$$\hat{\bar{Y}}_{\text{Re}g} = \bar{y} + b(\bar{X} - \bar{x}) = M_3(say) \quad \text{...(5)}$$

The *MSE* of the linear regression estimator up to first-degree approximation is

$$MSE(\hat{\bar{Y}}_{\text{Re}g}) = \phi\hat{\bar{Y}}^2C_y^2[1 - \rho_{yx}^2] \quad \text{...(6)}$$

Bahl & Tuteja (1991) has offered the product exponential-type estimators for the population mean. The product exponential-type estimator for the population mean

$$\hat{\bar{Y}}_{BT,Pe} = \bar{y}\exp\left(\frac{\bar{x} - \bar{X}}{\bar{x} + \bar{X}}\right) = M_4(say) \quad \text{...(7)}$$

The Bias of the product exponential-type estimator for the population mean is

$$Bias(\hat{\bar{Y}}_{BT,Pe}) = \frac{\phi}{2}\bar{Y}\left[\rho_{yx}C_yC_x - \frac{C_x^2}{4}\right] \quad \text{..................(8)}$$

The *MSE* of the product exponential-type estimator up to first-degree approximation for the population mean is

$$MSE(\hat{\bar{Y}}_{BT,Pe}) = \frac{\phi}{4}\bar{Y}^2[4C_y^2 + C_x^2 - 4\rho_{yx}C_yC_x] \quad \text{...} \quad (9)$$

(9)

…

Approaches to express the parameters

Shewhart-type control charts consist of three parameters: lower control limit (LCL), center line (CL), and upper control limit (UCL). We have two methods to define these parameters: the 3-sigma limits technique and the probability limit approach. For symmetric distributions, we employ the 3-sigma limit strategy, whereas for asymmetric distributions, we utilize the probability limit approach.

Probability Limits Approach

For the existing M_1 chart, we define probability approach as:

$LCL = M_{1l}$ With $p_n(M_1 = M_{1l}) \le \alpha_l$

$UCL = M_{1u}$ With $p_n(M_1 = M_{1u}) \ge 1-\alpha_u$

Where $\alpha = \alpha_1 + u$ and p_n be the cumulative distribution function for a current value of n.

$LCL = M_{1l} = M_1 + c_l\hat{\sigma}_y / \sqrt{n}$ with $p_n(C = C_1) \le \alpha_l$

$UCL = M_{1u} = M_1 + c_u\hat{\sigma}_y / \sqrt{n}$ with $p_n(C = C_u) \ge 1-\alpha_u$ (10)

Similarly, we will find probability limits approach for M_2, M_3 and M_4 respectively.

Sigma Limits Approach

The usual 3-sigma control limits with the parameters of the chart are given below:

$$\left.\begin{array}{l} LCL = M_1 - 3\sigma_{M1} \\ CL = M_1 \\ UCL = M_1 + 3\sigma_{M1} \end{array}\right\} (11)$$

By using Eq. (10) in Eq. (11), we get

$$\left.\begin{array}{l} LCL = M_1 - 3k_2\hat{\sigma}_y / \sqrt{n} \\ CL = M_1 \\ UCL = M_1 + 3k_2\hat{\sigma}_y / \sqrt{n} \end{array}\right\}$$

Where k_2 is known as standard error.

When selecting the structure of control charts for both the probability approach and the 3-sigma technique at a specific level of significance, we prioritize sample statistics that account for the temporal order of samples. Therefore, we conclude that if all samples fall within the control boundaries, it is evident that there is no shift in the process mean level, which remains stable over time. If the process mean is unstable, an assignable cause exists within the method, resulting in a change at the process mean level. Irfan el al. (2018) proposed family of estimators. Generalized class of difference-cum-exponential-type estimators of finite population mean is given below:

$$\hat{\bar{Y}}_p = \left[\frac{\bar{y}}{2}\left\{\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right) + \exp\left(\frac{\bar{x}-\bar{X}}{\bar{x}+\bar{X}}\right)\right\} + h_1(\bar{X}-\bar{x}) + h_2\bar{y}\right]\exp\left(\frac{\alpha(\bar{X}-\bar{x})}{\alpha(\bar{X}+\bar{x})-2\beta}\right) (12)$$

Where k_1 and k_2 are appropriate weights. $\alpha(\neq 0)$ and β can be the values of a known traditional and non-traditional parameters or a constant function of the secondary information x. ($\hat{\bar{Y}}_p$ For M_5 and M_6 (say)).

Special cases
By determining different constants or known constants additional variable population parameters in place of α and β in eq. (12), we can have various best estimators. In this research, we are using two of them, which are listed below

1. Case: I $\alpha = Q_D$ and $\beta = T_M$
2. Case: II $\alpha = Q_D$ and $\beta = M_R$

By using Eq. (13) in Eq. (12), the proposed estimators can be expressed as

$$\hat{\bar{Y}}_p = \left[\frac{\bar{y}(1+e_0)}{2}\left\{\exp\left(\frac{-e_1}{2}\left(1+\frac{e_1}{2}\right)^{-1}\right)+\exp\left(\frac{-e_1}{2}\left(1+\frac{e_1}{2}\right)^{-1}\right)\right\}-h_1\bar{X}e_1+h_2\bar{y}(1+e_0)\right]\exp\left(-\vartheta e_1(1+\vartheta e_1)^{-1}\right) \quad (13)$$

where

$$\vartheta = \frac{\alpha\bar{X}}{2(\alpha\bar{X}-\beta)}$$ is well-known measure.

Solving Eq. (13) up to first-degree approximation, we have

$$\hat{\bar{Y}}_p - \bar{Y} = \bar{Y}e_0 + \frac{\bar{Y}e_1^2}{8} - h_1\bar{X}e_1 + h_2\bar{Y} + h_2\bar{Y}e_0 - \vartheta\bar{Y}e_1 - \vartheta\bar{Y}e_0e_1 + \vartheta h_1\bar{X}e_1^2 - \vartheta h_2\bar{Y}e_1 - \vartheta h_2\bar{Y}e_0e_1 + \frac{3}{2}\vartheta^2\bar{Y}e_1^2 + \frac{3}{2}h_2\vartheta^2\bar{Y}e_1^2 \quad (14)$$

By applying expectations on both sides of above equation, the bias of suggested class of estimators is:

$$Bias(\hat{\bar{Y}}_p) \cong \frac{1}{8}[8h_2\bar{Y} + \phi C_x^2\{8h_1\bar{X}\vartheta + \bar{Y}(1+12\vartheta^2+12h_2\vartheta^2)\} - 8\bar{Y}\vartheta\phi C_x C_y(1+h_2)\rho_{yx}] \quad (15)$$

By squaring and applying expectations on both sides of Eq. (15) and after ignoring higher-order approximation, we have

$$MSE(\hat{\bar{Y}}_p) \cong \frac{1}{4}\phi C_x^2\left[4(\bar{Y}\vartheta+\bar{X}h_1)^2 + 4\bar{Y}\left\{\left(5\vartheta^2+\frac{1}{4}\right)\bar{Y} + 4\bar{X}h_1\vartheta\right\}h_2 + 16\bar{Y}^2\vartheta^2h_2^2\right] + \bar{Y}^2\left[h_2^2 + \phi C_y^2(1+h_2)^2\right] \\ -2\bar{Y}\phi\rho_{yx}C_xC_y\left[\left(1+h_2\right)\left(\bar{Y}\vartheta+\bar{X}h_1+2\bar{Y}\vartheta h_2\right)\right]$$
(16)

By differentiating Eq. (16) with respect to h_1, we get

The optimal value of h_1 is:

$$h_{1(opt)}=\frac{\bar{Y}\left[-2\rho_{yx}C_y+C_x\left\{2\vartheta-2\phi\vartheta C_x^2\left(\vartheta^2+\frac{1}{4}\right)+\phi\rho_{yx}C_yC_x\left(\vartheta^2+\frac{1}{4}\right)+2\vartheta\phi\left(-1+\rho_{yx}^2\right)C_y^2\right\}\right]}{2\bar{X}C_x\left[-1+\phi C_y^2\left(-1+\rho_{yx}^2\right)\right]}$$

By differentiating Eq. (16) with respect to h_2 and put the equation equal to zero

The optimal value of h_2 is:

$$h_{2(opt)}=\frac{\phi\left[C_x^2\left(\vartheta^2+\frac{1}{4}\right)-2C_y^2\left(-1+\rho_{yx}^2\right)\right]}{4\left[-1+\phi C_y^2\left(-1+\rho_{yx}^2\right)\right]}$$

By putting optimal values of h_1 and h_2 in Eq. (17), we can get the minimum MSE of the proposed generalized class of estimators as:

$$MSE_{\min}(\hat{\hat{\bar{Y}}}_p)=\frac{\phi\bar{Y}^2\left[\phi C_x^4\left(\vartheta^2+\frac{1}{4}\right)-4\left(-1+\rho_{yx}^2\right)\left(-1+\phi C_x^2\left(\vartheta^2+\frac{1}{4}\right)C_y^2\right)\right]}{4\left[-1+\phi C_y^2\left(-1+\rho_{yx}^2\right)\right]} \quad (17)$$

After interpretations, the modified form of above equation can be written as:

$$MSE_{\min}(\hat{\hat{\bar{Y}}}_p)\cong MSE(\hat{\hat{\bar{Y}}}_{\text{Re}g})-[R_1+R_2] \quad (18)$$

Where

$$R_1=\frac{\phi^2\bar{Y}^2\left[C_x^2\left(\vartheta^2+\frac{1}{4}\right)+4\left(1-\rho_{yx}^2\right)C_y^2\right]^2}{16\left[1+\phi C_y^2\left(1-\rho_{yx}^2\right)\right]}$$

And

$$R_2=\frac{\phi^2\bar{Y}^2C_x^2\left[3C_x^2\left(\vartheta^2+\frac{1}{4}\right)+8\left(1-\rho_{yx}^2\right)\left(\vartheta^2+\frac{1}{4}\right)C_y^2\right]^2}{16\left[1+\phi C_y^2\left(1-\rho_{yx}^2\right)\right]}$$

Both are always must be positive measures.

Simulation study

This study employs computerized numerical simulations to determine the appropriate control charts for enhancing the process average. The Monte Carlo simulation approach is employed to compute the ARL values corresponding to various shift magnitudes. Random numbers have been created utilizing the R console statistical software. We have utilized many statistical packages, including bivariate and multivariate normal distribution packages, employing R software.

Simulations Details

Simulation Steps For In-control ARL_0 And Out-of-control ARL_1

If the in-control ARL_0 and out-of-control ARL_1 are likely to be known during simulation method, then the steps to the simulation process are given below:

1. An out-of-control sample of 100000 is generated according to the specified distribution limits by generating multivariate normal random numbers.
2. By fixing the value of ARL at 371.
3. We fixed the value of correlation coefficient at ρ_{yx}=0.3, ρ_{yx}=0.6, ρ_{yx}=0.90.

Consider $alpha = \frac{1}{ARL_0}$

4. Fixed the lower and upper probability point.
5. Lower Probability Point is $LProb = \frac{alpha}{2}$.
6. Upper Probability Point is $U\Pr ob = \frac{1-alpha}{2}$.
7. Standardized the mean and variance of multivariate normal distribution with mean =10 and variance = 1.
8. Construct existing and proposed estimator in term of finding control limit.
9. Now generate Large random number (sim=100000) for finding control limits.
10. Compute the upper and lower control limit (LCL, UCL) for existing and proposed estimators.
11. Now calculate out-of-control ARL for sim with shift as δ = 0.0, 0.25, 0.50, 0.75, 1.00, 1.50, 2.00, 2.50 and 3.00.
12. Based on the control chart criteria, it is determined whether this sample results in an out-of-control signal.
13. If the sample output results in an out-of-control indication, than the sample number is make a note as the run length for that simulation and if the sample does not result in an out-of-control signal, returns to Step 1.
14. Steps 1-4 are repeated until the number of simulations (sim) is reached. The result is sim run lengths.
15. The normal or specified quantile of the run length distribution is reported.

Simulation Steps For In-control Mean and In-Control Standard deviation

Suppose the in-control mean and in-control standard deviations are to be simulated based on in-control initial samples (sim). In that case, the steps to the simulation process are as follows (assume a sample consists of n observations).

1. Sim in-control samples of size n are created according to the specified distribution parameters of the in-control distribution.

2. The in-control average and standard deviation are calculated based on the simulated in-control samples.
3. An out-of-control sample of size n is produced according to the definite distribution parameters of the out-of-control distribution.
4. The average of the sample is produced and, if necessary for the specific type of control chart, the standard deviation is also provided.
5. Based on the control chart criteria, it is determined whether this sample results in an out-of-control signal.
6. If the sample results in an out-of-control signal, the sample number is recorded as the run length for that simulation. If the sample does not result in an out-of-control signal, return to Step 3.
7. Steps 1 through 6 are repeated until the number of simulations (sim) is reached. The result is the simulation run lengths.
8. The average, median, or specified percentile of the run length distribution is reported.
9. Once control charts are established, it is crucial to understand their interpretation and use in the event of an issue. The control charts are separated into three categories. The initial segment is termed "out of statistical control" for several reasons. Ensure that points that do not coincide on the chart are situated outside the control limits. This indicates that specific causes may alter. The occurrence of points beyond the regulatory limits is typically the most conspicuous circumstance. In the second segment, the process is evaluated, albeit ineffectively. All points are under control, indicating a singular cause. In the third segment, it will be noted that trends are more predictable and exhibit a smoother progression. This section demonstrates evidence of process enhancement and a reduction in variation.

Performance measures and comparisons

We may assess the performance and assessments of ARL to achieve optimal and efficient outcomes by employing both conventional and unconventional metrics in two manners: Shift-to-Shift Performance Measure and Overall Performance Measure. Monte Carlo simulation is a statistical computational technique that depends on repeated random sampling to derive numerical outcomes. Average Run Length is defined as the mean number of samples necessary for a control chart to indicate an out-of-control condition. Extra Quadratic Loss (EQL) is a metric that delineates the overall efficacy of a control chart. Relative Average Run Length (RARL) is a metric utilized to assess the performance of control charts comprehensively and to compare several charts within the shift range. The Performance Comparison Index (PCI) is defined as the ratio of the EQL to the EQL_benchmark, where the benchmark is the minimum value of EQL in the control charts.

The average run length results of Shewhart chart when the in-control ARL

Table 1: Average Run length of data set 1 at ARL=371, n=5 and $\rho_{yx} = 0.30$

Shift	M_1	M_2	M_3	M_4	M_5	M_6
0.00	384.44	360.90	378.91	361.76	362.81	362.81
0.25	146.81	224.56	157.68	132.23	152.61	152.61
0.50	35.74	80.80	39.49	31.45	41.72	41.72
0.75	10.75	29.21	13.16	9.96	13.16	13.16
1.00	4.54	13.88	5.05	4.07	5.65	5.65
1.50	1.59	4.17	1.61	1.44	1.71	1.71
2.00	1.08	1.88	1.09	1.05	1.12	1.12
2.50	1.00	1.23	1.01	1.00	1.01	1.01
3.00	1.00	1.04	1.00	1.00	1.00	1.00

LCL	3.64	3.60	3.65	3.73	3.67	3.67
UCL	6.35	6.93	6.37	6.30	6.37	6.37

Table 2: Average Run length of data set 2 at ARL=371, n=5 and $\rho_{yx} = 0.60$

Shift	M_1	M_2	M_3	M_4	M_5	M_6
0.00	384.44	360.90	367.82	359.11	361.36	361.36
0.25	146.81	158.71	238.69	102.78	114.66	114.66
0.50	35.74	36.35	70.19	20.06	26.05	26.05
0.75	10.75	12.85	18.79	6.14	8.06	8.06
1.00	4.54	5.52	6.37	2.60	3.27	3.27
1.50	1.59	1.76	1.60	1.14	1.29	1.29
2.00	1.08	1.12	1.05	1.00	1.02	1.02
2.50	1.00	1.01	1.01	1.00	1.00	1.00
3.00	1.00	1.00	1.00	1.00	1.00	1.00
LCL	3.64	3.90	3.62	3.93	3.87	3.88
UCL	6.35	6.40	6.37	6.10	6.17	6.17

Table 3: Average Run Length of data set 3 at ARL=371, n=5 and $\rho_{yx} = 0.90$

Shift	M_1	M_2	M_3	M_4	M_5	M_6
0.00	384.44	360.90	351.56	368.65	347.73	347.73
0.25	146.81	30.64	151.24	27.42	43.89	43.89
0.50	35.74	4.47	18.56	3.39	5.66	5.66
0.75	10.75	1.55	3.27	1.26	1.73	1.73
1.00	4.54	1.05	1.35	1.02	1.11	1.11
1.50	1.59	1.00	1.00	1.00	1.00	1.00
2.00	1.08	1.00	1.00	1.00	1.00	1.00
2.50	1.00	1.00	1.00	1.00	1.00	1.00
3.00	1.00	1.00	1.00	1.00	1.00	1.00
LCL	3.64	4.42	4.11	4.11	4.40	4.40
UCL	6.35	5.67	5.85	5.61	5.70	5.70

Table 4: Average Run Length of data set 4 at ARL=371, n=10 and $\rho_{yx} = 0.30$

Shift	M_1	M_2	M_3	M_4	M_5	M_6
0.00	355.91	342.10	342.52	335.49	363.29	363.29
0.25	67.85	123.68	65.74	59.54	71.16	71.16
0.50	12.09	29.54	10.60	9.83	11.74	11.74
0.75	3.72	9.05	3.27	3.22	3.64	3.64
1.00	1.71	4.10	1.59	1.54	1.67	1.67
1.50	1.04	1.39	1.02	1.01	1.03	1.03
2.00	1.00	1.03	1.00	1.00	1.00	1.00
2.50	1.00	1.00	1.00	1.00	1.00	1.00
3.00	1.00	1.00	1.00	1.00	1.00	1.00
LCL	3.64	3.60	3.65	3.73	3.66	3.67
UCL	6.35	6.93	6.37	6.30	6.36	6.36

Table 5: Average Run Length of data set 5 at ARL=371, n=10 and $\rho_{yx} = 0.60$

Shift	M_1	M_2	M_3	M_4	M_5	M_6
0.00	355.91	347.43	354.57	333.28	354.96	354.96
0.25	67.85	72.74	48.20	42.98	45.21	45.21
0.50	12.09	12.66	7.27	6.30	6.79	6.79
0.75	3.72	3.86	2.20	2.02	2.11	2.11
1.00	1.71	1.81	1.21	1.16	1.22	1.22
1.50	1.04	1.03	1.00	1.00	1.00	1.00
2.00	1.00	1.00	1.00	1.00	1.00	1.00
2.50	1.00	1.00	1.00	1.00	1.00	1.00
3.00	1.00	1.00	1.00	1.00	1.00	1.00
LCL	3.64	3.90	3.62	3.92	3.87	3.87
UCL	6.35	6.40	6.37	6.10	6.17	6.17

Table 6: Average Run Length of data set 6 at ARL=371, n=10 and $\rho_{yx} = 0.90$

Shift	M_1	M_2	M_3	M_4	M_5	M_6
0.00	355.91	345.94	369.41	357.48	342.10	342.10
0.25	67.85	11.45	12.10	8.86	9.39	9.39
0.50	12.09	1.57	1.56	1.36	1.49	1.49
0.75	3.72	1.01	1.01	1.01	1.01	1.01
1.00	1.71	1.00	1.00	1.00	1.00	1.00
1.50	1.04	1.00	1.00	1.00	1.00	1.00
2.00	1.00	1.00	1.00	1.00	1.00	1.00
2.50	1.00	1.00	1.00	1.00	1.00	1.00
3.00	1.00	1.00	1.00	1.00	1.00	1.00
LCL	4.04	4.58	4.55	4.57	4.57	4.57
UCL	5.39	5.44	5.44	5.41	5.42	5.42

Table 7: Average Run Length of data set 7 at ARL=371, n=15 and $\rho_{yx} = 0.30$

Shift	M_1	M_2	M_3	M_4	M_5	M_6
0.00	383.72	364.44	376.50	379.99	357.40	357.40
0.25	49.38	79.88	46.12	42.83	40.16	40.16
0.50	7.02	15.57	6.06	5.92	5.92	5.92
0.75	2.22	4.52	1.87	1.82	1.86	1.86
1.00	1.25	2.08	1.16	1.16	1.16	1.16
1.50	1.00	1.07	1.00	1.00	1.00	1.00
2.00	1.00	1.00	1.00	1.00	1.00	1.00
2.50	1.00	1.00	1.00	1.00	1.00	1.00
3.00	1.00	1.00	1.00	1.00	1.00	1.00
LCL	4.04	3.98	4.08	4.09	4.05	4.05
UCL	5.93	6.26	5.90	5.89	5.90	5.90

Table 8: Average Run Length of data set 8 at ARL=371, n=15 and $\rho_{yx} = 0.60$

Shift	M_1	M_2	M_3	M_4	M_5	M_6
0.00	383.72	365.87	363.08	362.14	371.83	371.83
0.25	49.38	41.81	27.96	26.90	25.80	25.80
0.50	7.02	6.13	3.55	3.40	3.53	3.53
0.75	2.22	2.01	1.32	1.32	1.37	1.37
1.00	1.25	1.16	1.04	1.02	1.04	1.04
1.50	1.00	1.00	1.00	1.00	1.00	1.00
2.00	1.00	1.00	1.00	1.00	1.00	1.00
2.50	1.00	1.00	1.00	1.00	1.00	1.00
3.00	1.00	1.00	1.00	1.00	1.00	1.00
LCL	4.04	4.20	4.20	4.24	4.21	4.21
UCL	5.93	5.93	5.77	5.75	5.76	5.76

Table 9: Average Run Length of data set 9 at ARL=371, n=15 and $\rho_{yx} = 0.90$

Shift	M_1	M_2	M_3	M_4	M_5	M_6
0.00	383.72	367.51	354.02	368.59	353.34	353.34
0.25	49.38	5.35	4.89	4.43	4.66	4.66
0.50	7.02	1.12	1.09	1.06	1.09	1.09
0.75	2.22	1.00	1.00	1.00	1.00	1.00
1.00	1.25	1.00	1.00	1.00	1.00	1.00
1.50	1.00	1.00	1.00	1.00	1.00	1.00
2.00	1.00	1.00	1.00	1.00	1.00	1.00
2.50	1.00	1.00	1.00	1.00	1.00	1.00
3.00	1.00	1.00	1.00	1.00	1.00	1.00
LCL	4.04	4.58	4.55	4.57	4.57	4.57
UCL	5.93	5.44	5.44	5.41	5.42	5.42

Table 10: Shift-to-Shift Performance Table(EQL, RARL and PCI values for ARL=371)

N	ρ	Measures	M_1	M_2	M_3	M_4	M_5	M_6
5	0.3	EQL	7.4427	11.8982	7.7119	7.1821	7.8338	7.8338
		RARL	1.0515	1.9361	1.0965	1.0000	1.1250	1.1250
		PCI	1.0363	1.6566	1.0738	1.0000	1.0907	1.0907
	0.6	EQL	7.4427	7.7647	8.7969	6.4952	6.8227	6.8227
		RARL	1.2591	1.3515	1.5666	1.0000	1.0900	1.0900
		PCI	1.1459	1.1955	1.3544	1.0000	1.0504	1.0504
	0.9	EQL	7.4427	5.6237	6.4036	5.5813	5.7060	5.7060
		RARL	2.7529	1.0437	1.6905	1.0000	1.1092	1.1092
		PCI	1.3335	1.0076	1.1473	1.0000	1.0224	1.0224
10	0.3	EQL	6.0375	7.0607	5.9733	5.9278	6.0356	6.0356
		RARL	1.0488	1.5149	1.0173	1.0000	1.0455	1.0455
		PCI	1.0185	1.1911	1.0077	1.0000	1.0182	1.0182
	0.6	EQL	6.0375	6.0770	5.7739	5.7273	5.7525	5.7525

		RARL	1.1978	1.2209	1.0288	1.0000	1.0178	1.0178
		PCI	1.0542	1.0611	1.0081	1.0000	1.0044	1.0044
		EQL	6.0375	5.4798	5.4821	5.4664	5.4705	5.4705
	0.9	RARL	2.1484	1.0269	1.0331	1.0000	1.0084	1.0084
		PCI	1.1045	1.0025	1.0029	1.0000	1.0008	1.0008
		EQL	5.7791	6.2102	5.7306	5.7138	5.7048	5.7048
	0.3	RARL	1.0476	1.3368	1.0128	1.0048	1.0000	1.0000
		PCI	1.0130	1.0886	1.0045	1.0016	1.0000	1.0000
		EQL	5.7791	5.7198	5.5898	5.5815	5.5829	5.5829
15	0.6	RARL	1.1844	1.1307	1.0071	1.0000	1.0049	1.0049
		PCI	1.0354	1.0248	1.0015	1.0000	1.0002	1.0002
		EQL	5.7791	5.4486	5.4463	5.4440	5.4454	5.4454
	0.9	RARL	2.0866	1.0164	1.0070	1.0000	1.0037	1.0037
		PCI	1.0615	1.0008	1.0004	1.0000	1.0003	1.0003

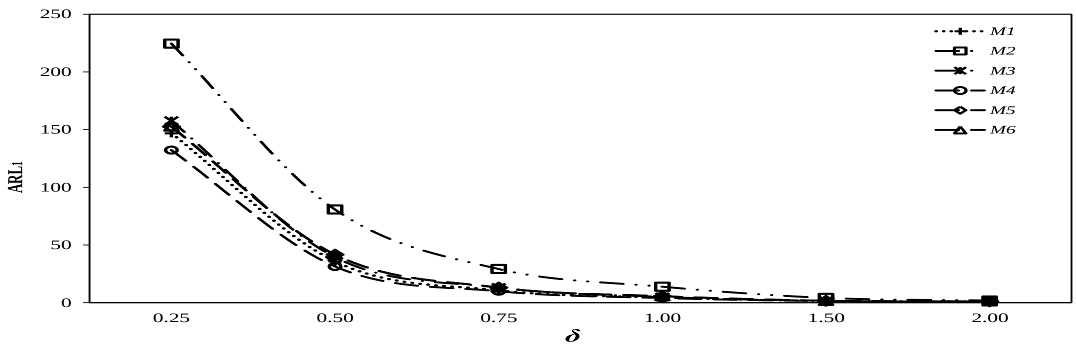

Figure 1: *ARL* Comparisons of $M_1 - M_6$charts for $n = 5$ and$\rho_{yx} = 0.30$.

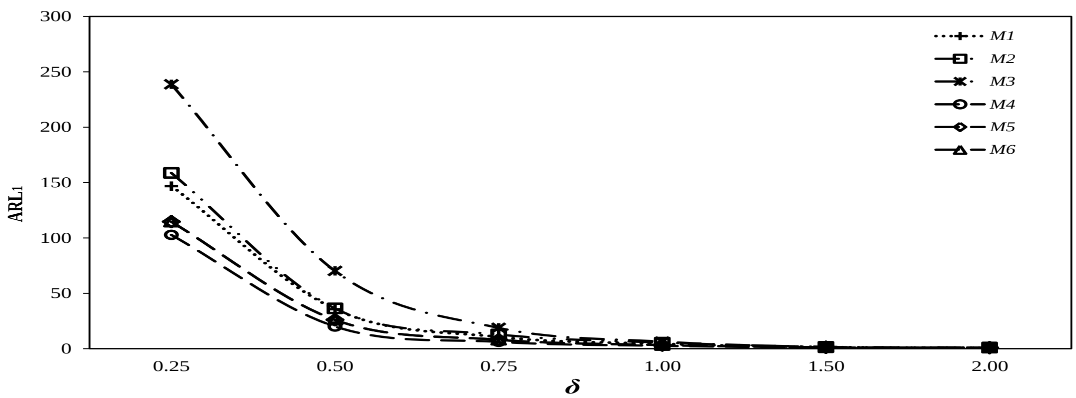

Figure.2: *ARL* Comparisons of $M_1 - M_6$ charts for $n = 5$ and$\rho_{yx} = 0.60$

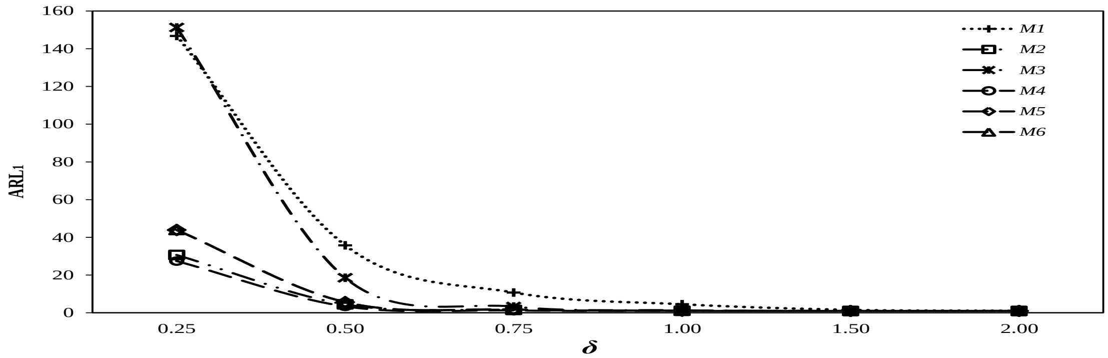

Figure 3: ***ARL*** **Comparisons of $M_1 - M_6$charts for $n = 5$ and$\rho_{yx} = 0.90$**

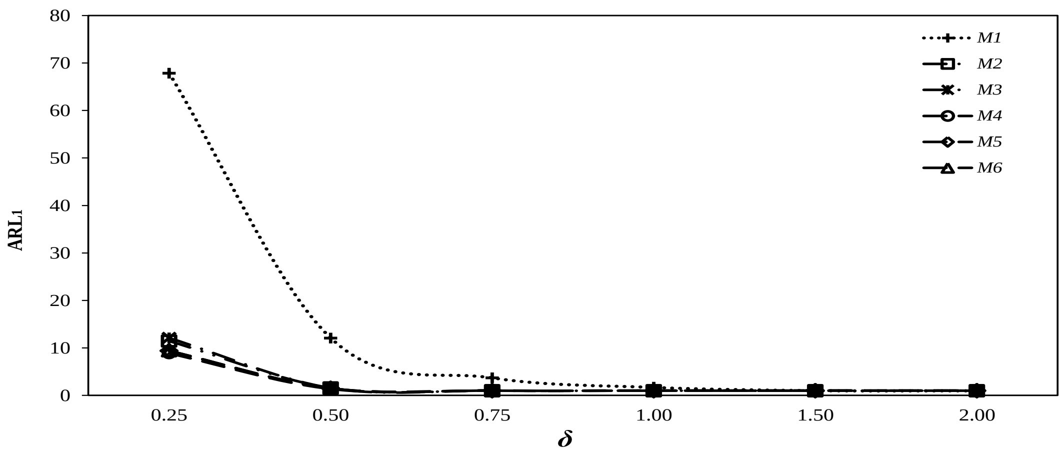

Figure 4: ***ARL*** **Comparisons of $\mathrm{M}_1 - \mathrm{M}_6$charts for $n = 10$ and$\rho_{yx} = 0.30$.**

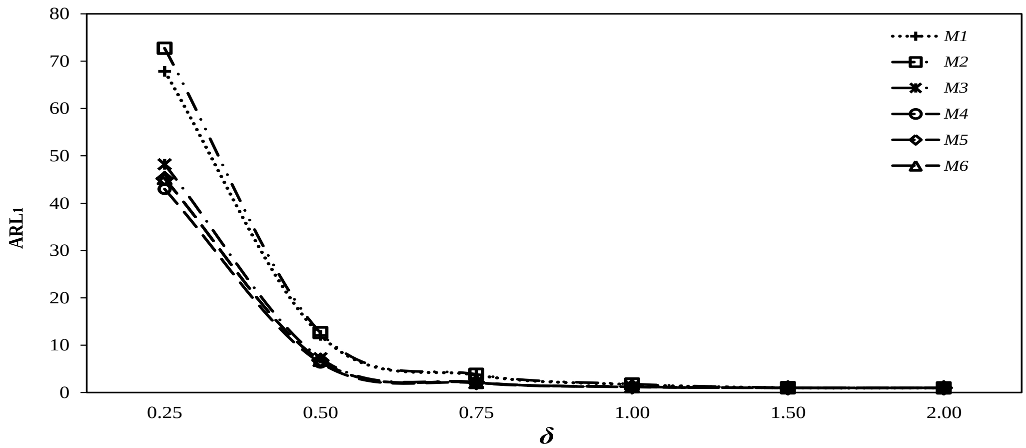

Figure 5: ***ARL*** **Comparisons of $M_1 - M_6$ charts for $n = 10$ and$\rho_{yx}0.60$.**

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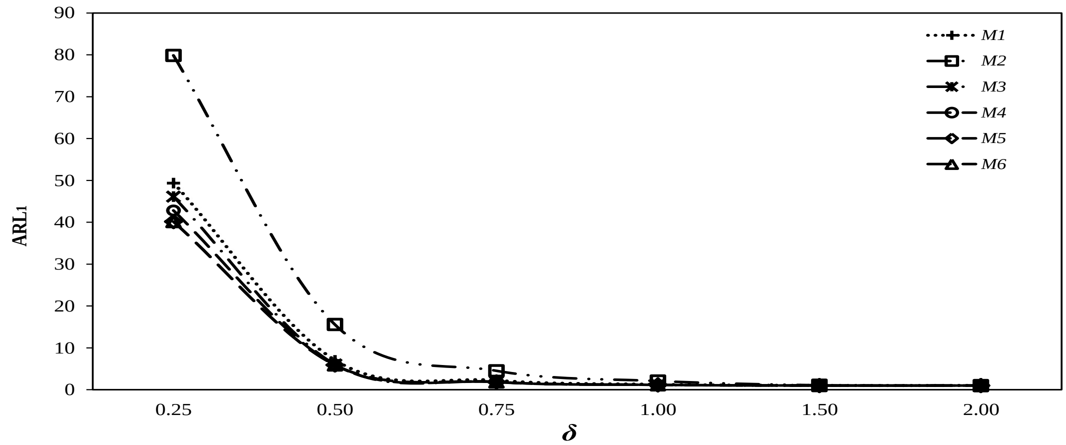

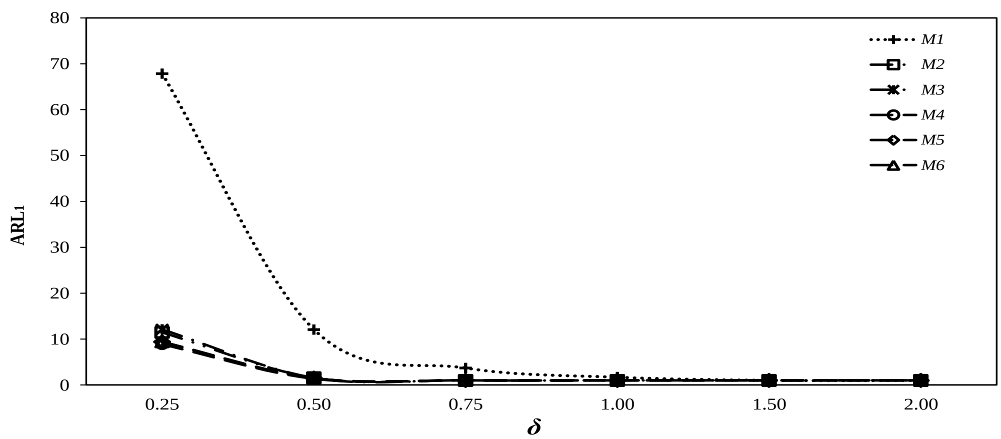

Figure 6: ***ARL*** **Comparisons of** $M_1 - M_6$**charts for** $n = 10$ **and**$\rho_{yx} = 0.90$**.**

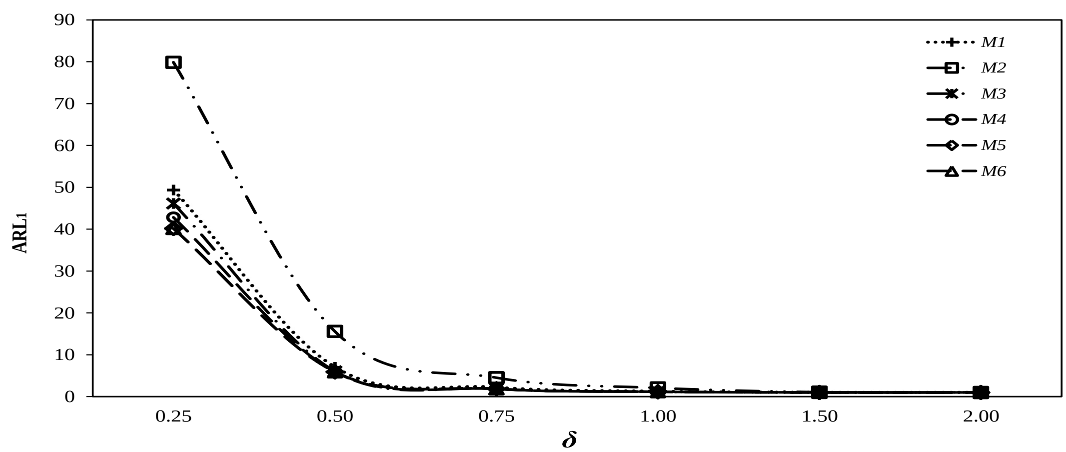

Figure 7: ***ARL*** **Comparisons of** $M_1 - M_6$ **charts for** $n = 15$ **and**$\rho_{yx}0.30$**.**

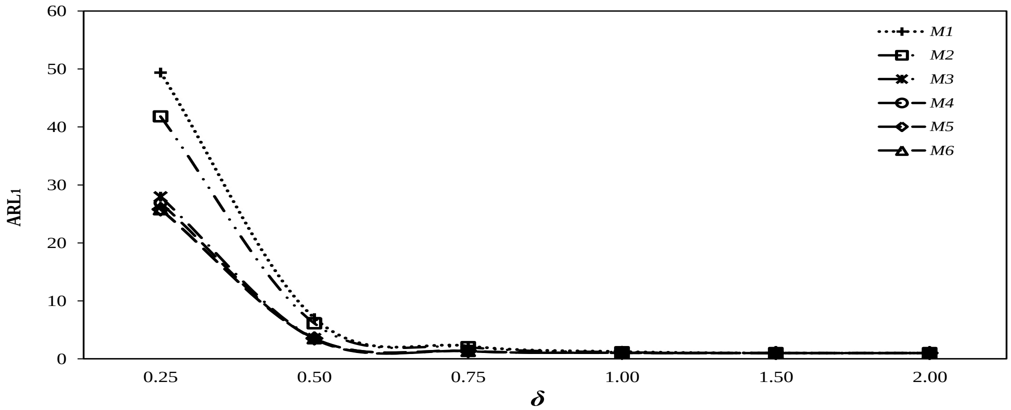

Figure 8: ARL Comparisons of $M_1 - M_6$ charts for$n = 15$and$\rho_{yx}0.60$.

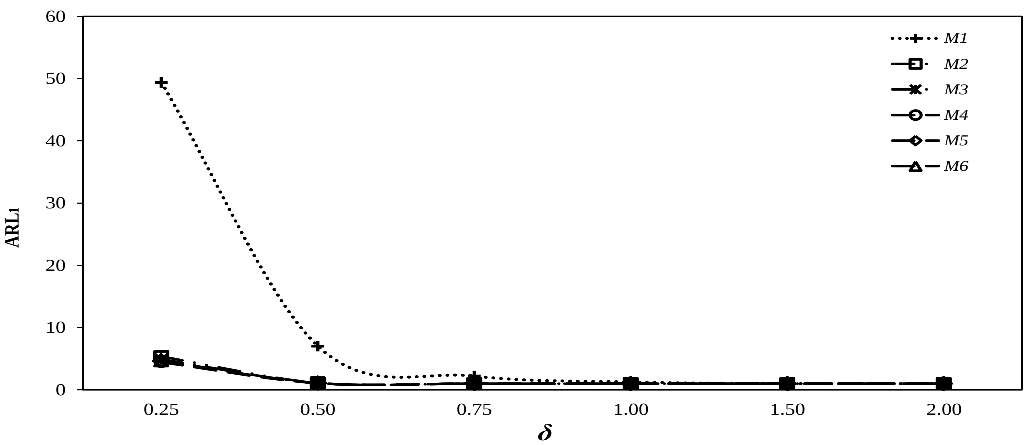

Figure 9: ARL Comparisons of $M_1 - M_6$ charts for $n = 15$ and$\rho_{yx} = 0.90$.

Based on the results in Tables 1-10 and Figs. 1-9, we summarize our major findings from the Shewhart charts as follow:

Tables (1-9) shows that M_1, M_2, M_3, M_4, M_5 and M_6 estimators provides the average run length for different shift levels**.** By fixing value ARL =371, n=5, 10 and 15 and$\rho_{yx} = 0.30, 0.60$ *and* 0.90, we can say that the *ARL's* of the estimators M_5 and M_6 are fewer than other estimators. Therefore, most effectual results are attained at these estimators.

Table 10 shows the properties of estimation of the control limits on the presentation of the current Shewhart $\overline{X}$ chart are studied through the average run length when both the process mean and variance are known. We use different shifts level at different values of sample size by fixing *ARL* at 371. By keeping value at n=5,$\rho_{yx} = 0.60$**,**the value of M_1 estimator has *EQL*=7.4427 which gradually decreased at M_6 estimator having *EQL*=6.8227. Likewise, by increasing sample size, the *EQL* measure has decreased M_6 at estimator. At n=15 and $\rho_{yx} = 0.90$, the value of *PCI*=1.0615 at M_1 whereas at M_6 the value of *PCI*=1.0003 which shows its decreasing trend. Similarly, *PCI* and *RARL* measures have the same descending case by increasing sample sizes at different levels as well as with the increase of correlation coefficient. The shift-to-shift performance shows that by increasing shifts level as well as by increasing correlation coefficient, we obtained best *ARL*, *PCI* and *RARL* results. Hence, showing that M_6 is the most efficient estimator among all estimators. 3.1 Figure (1-9) based on bivariate normal distribution when $ARL = 371$, $n = 5, 10$ and 15 and $\rho_{yx} = 0.30, 0.60$ and 0.90, we have designed average run length of M_1, M_2, M_3, M_4, M_5 and M_6 estimators against different values of shifts, we detected that our robust estimators are more proficient than other traditional estimators.

Illustrative example

In order to illustrate the application of the control structures under study and highlight their importance in an efficient detection of changes in process parameters, we provide here descriptive examples to compare the performance of the structures of M_1 vsM_6. We conclude the process has shifted and is unstable if we observe a point outside the control limits. Among all estimators M_6 is the most efficient estimator so we are comparing it with the usual mean estimatorM_1. For the said purpose, we have generated data sets containing 50 subgroups using sample sizes n=5, 10, 15 from the bivariate normal distribution. The first 30 observations are generated from the in-control situation, i.e.,$\delta = 0$, whereas the remaining 20 observations are generated from an out-of-control situation with $\delta = 1$ for both data sets. By using these values ($\mu_y = \mu_x = 2; \sigma_y = \sigma_x = 1$; Shift $\delta = 0.60$, $n = 15$ and$\rho_{yx} = 0.90$) we conclude that M_1 detected shift in 3 sub-groups while M_6 detected shift in 12 sub-groups. As M_6 chart detected more out-of-control signals than M_1 chart, so it is justified that M_6 chart has better detection ability than M_1

Table 11: Control Charting Values of M_1 to M_6Charts

Seri No	Serial	M_1	M_2	M_3	M_4	M_5	M_6
1		1.5891	2.1807	2.1342	2.1649	2.1177	2.1177
2		1.9521	1.8687	1.9286	1.9002	1.8855	1.8855
3		2.2192	1.7960	1.7666	1.8175	1.7661	1.7661
4		2.0530	2.0481	2.0416	2.0428	2.0119	2.0119
5		2.3617	1.9821	1.9585	1.9531	1.9515	1.9515
6		1.8722	1.9151	1.9143	1.9459	1.9471	1.9471
7		2.0069	1.9627	1.8925	1.9220	1.9467	1.9467
8		2.1779	2.0026	2.0069	2.0106	1.9863	1.9863
9		1.8903	2.0031	1.9999	2.0240	2.0217	2.0217
10		1.7785	1.9900	1.9981	1.9761	1.9542	1.9542
11		2.3157	2.1675	2.1546	2.1620	2.1239	2.1239
12		1.7473	1.8558	1.8584	1.8647	1.8294	1.8294
13		1.7506	1.8900	1.8984	1.9074	1.8768	1.8768
14		1.8277	1.9752	1.9766	1.9758	1.9478	1.9478
15		1.6123	1.9120	1.9351	1.9405	1.9341	1.9341
16		1.8106	2.0390	2.0392	2.0328	2.0025	2.0025
17		1.9313	2.0853	2.0877	2.0871	2.0593	2.0593
18		1.9898	2.1647	2.1247	2.1367	2.0916	2.0916
19		1.4155	1.9192	1.9176	1.9477	1.9437	1.9437
20		1.6388	2.0821	2.0875	2.0885	2.0667	2.0667
21		1.8878	2.0184	2.0319	2.0257	2.0019	2.0019

22	1.6935	2.0498	2.0604	2.0684	2.0685	2.0685
23	2.1434	1.8343	1.8404	1.8381	1.8066	1.8066
24	1.8934	1.9663	1.9632	1.9722	1.9447	1.9447
25	1.9817	2.0529	1.9839	1.9748	2.0340	2.0340
26	2.0370	2.1893	2.1589	2.1807	2.1378	2.1378
27	1.8679	2.1848	2.1935	2.1879	2.1629	2.7729
28	1.6384	2.0750	2.1041	2.1032	2.1443	2.7443
29	1.6885	1.9779	1.9596	2.0162	2.0776	2.6776
30	2.1070	2.2637	2.2105	2.2352	2.1695	2.1695
31	2.4374	2.3689	2.4329	2.4098	2.6308	2.6308
32	2.7462	2.4534	2.6053	2.5035	2.8646	2.8646
33	2.1380	2.6176	2.6854	2.6393	2.7334	2.7334
34	2.8821	2.4752	2.3972	2.4393	2.3243	2.3243
35	2.8427	2.6973	2.5797	2.6489	2.4677	2.4677
36	2.6500	2.7121	2.7275	2.7178	2.7104	2.7104
37	2.8068	2.5522	2.5017	2.5252	2.4089	2.8089
38	2.0065	2.4960	2.5195	2.5024	2.4945	2.4945
39	2.1960	2.6595	2.6718	2.6627	2.6415	2.6415
40	2.7164	2.6463	2.4858	2.5832	2.4011	2.4011
41	2.2042	2.6271	2.6859	2.6409	2.6800	2.6800
42	2.4343	2.3956	2.5101	2.4309	2.6027	2.6027
43	3.0908	2.5529	2.6278	2.5741	2.6573	2.6573
44	2.6755	2.5186	2.6354	2.5590	2.8130	2.8130
45	2.8221	2.6608	2.6391	2.6552	2.5919	2.5919
46	2.7643	2.6717	2.6607	2.6612	2.5778	2.5778
47	2.4551	2.4632	2.4757	2.4689	2.4568	2.4568
48	2.5542	2.3895	2.4021	2.4064	2.4446	2.6446
49	2.6957	2.4396	2.5522	2.5099	3.2574	3.2574
50	2.2904	2.6189	2.5170	2.5878	2.4524	2.4524

In Table 11, we are given 50 generated values of some estimators i.e. M_1, M_2, M_3, M_4, M_5 and M_6. The average run length of traditional estimators are compared with the robust estimators. The robust estimators detect much more outliers in the data sets. By way of M_6 chart detected more out-of-control signals than M_1 chart, so it is justified that M_6 chart has better detection ability than M_1. Therefore, traditional estimators perform less effectively than the robust estimators.

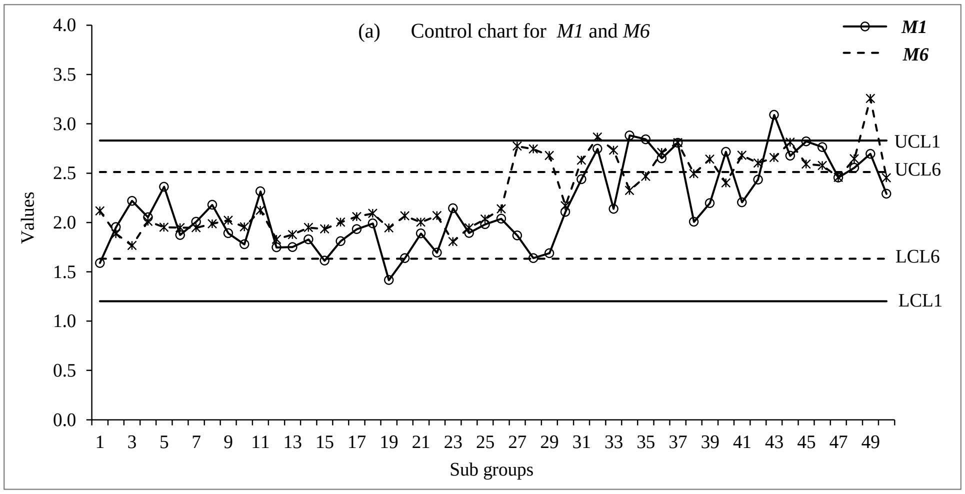

Figure 10: Control Charting Display of M_1 vs. M_6 charts

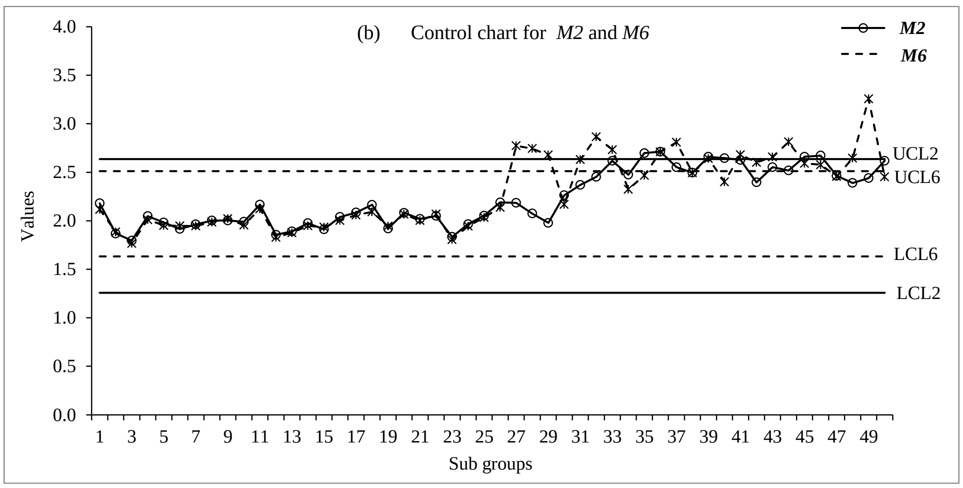

Figure 11: Control Charting Display of M_2 vs. M_6 charts

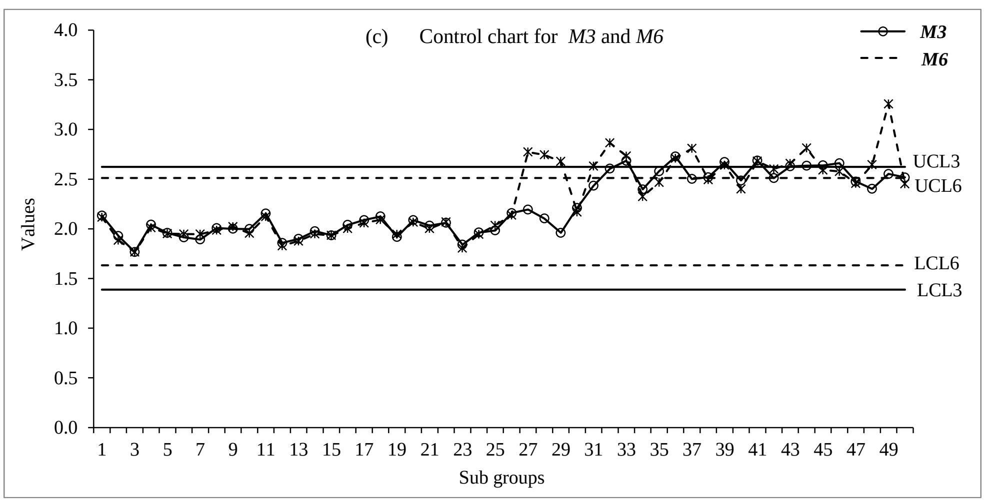

Figure 12: **Control Charting Display of M_3 vs. M_6 charts**

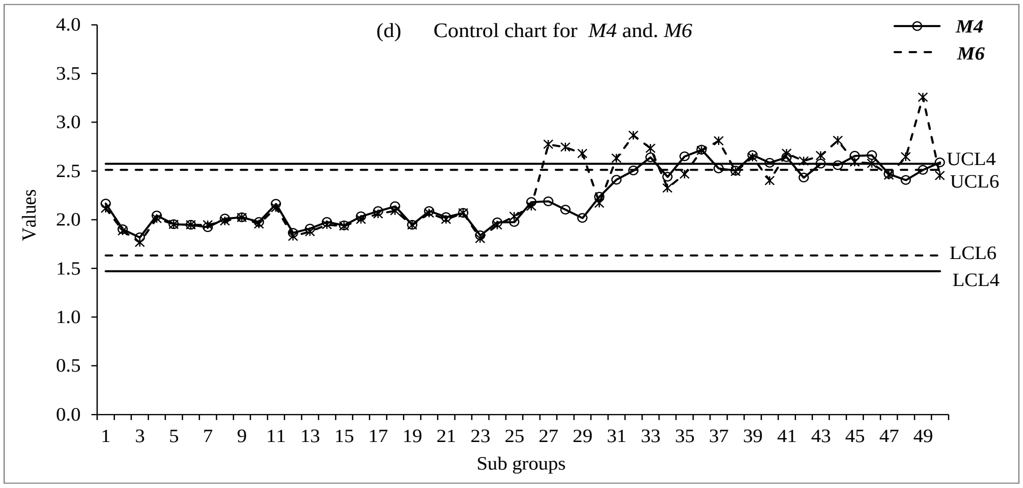

Figure13: Control Charting Display of M_4 vs. M_6 charts

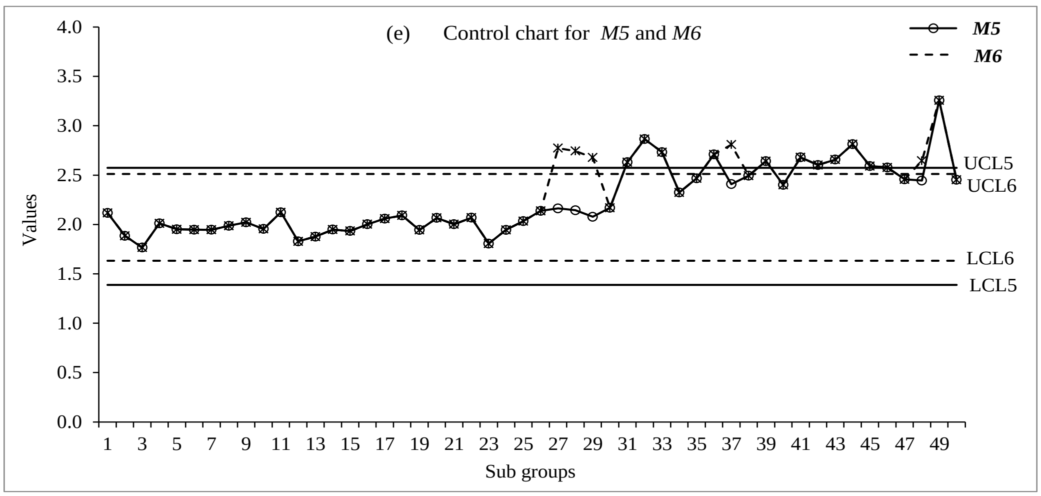

Figure14: Control Charting Display of M_5 vs. M_6 charts

In Figure 10, we can say that by using these values ($\mu_y = \mu_x = 2;\sigma_y = \sigma_x = 1$; Shift $\delta = 0.60$, $n = 15$ and $\rho_{yx} = 0.90$) we conclude that M_1 detected shift in 3 sub-groups while M_6 detected shift in 17 sub-groups. As M_6 chart detected 14 more out-of-control signals than M_1 chart, so it is justified that M_6 chart has better detection ability than M_1. In Figure 11, we can say that by using these values ($\mu_y = \mu_x = 2;\sigma_y = \sigma_x = 1$; Shift $\delta = 0.60$, $n = 15$ and $\rho_{yx} = 0.90$) we conclude that M_2 detected shift in 6 sub-groups while M_6 detected shift in 17 sub-groups. As M_6 chart detected 11 more out-of-control signals than M_2 chart, so it is justified that M_6 chart has better detection ability than M_2. In Figure 12, we can say that by using these values ($\mu_y = \mu_x = 2;\sigma_y = \sigma_x = 1$; Shift $\delta = 0.60$, $n = 15$ and $\rho_{yx} = 0.90$) we conclude that M_3 detected shift in 8 sub-groups while M_6 detected shift in 17 sub-groups. As M_6 chart detected 9 more out-of-control signals than M_3 chart, so it is justified that M_6 chart has better detection ability than M_3. In Figure 13, we can say that by using these values ($\mu_y = \mu_x = 2; \sigma_y = \sigma_x = 1$; Shift $\delta = 0.60$, $n = 15$ and $\rho_{yx} = 0.90$) we conclude that M_4 detected shift in 10 sub-groups while M_6 detected shift in 17 sub-groups. As M_6 chart detected 7 more out-of-control signals than M_4 chart, so it is justified that M_6 chart has better detection ability than M_4. In Figure 14, we can say that by using these values ($\mu_y = \mu_x = 2; \sigma_y = \sigma_x = 1$; Shift $\delta = 0.60$, $n = 15$ and $\rho_{yx} = 0.90$) we conclude that M_5 detected shift in 12 sub-groups while M_6 detected shift in 17 sub-groups. As M_6 chart detected 5 more out-of-control signals than M_5 chart, so it is justified that M_6 chart has better detection ability than M_5.

GENERAL DISCUSSION

We summarize the above results in the following steps:

- The measure of performance used is the average run length (*ARL*).
- The results show that the ability of the Shewhart charts to detect shifts in the process mean is quite.
- Robust to data correlation, while the corresponding individuals Shewhart charts rarely detects these shifts more quickly than the other charts.
- When sample size and correlation both are increased, we get efficient/better results.

SUMMARY, CONCLUSION AND RECOMMENDATION

Every industrial production process is subject to two distinct types of variation: common-cause variation and special-cause variation. Common-cause variation represents the inherent, random fluctuations that are part of the regular operation of a process. In contrast, special-cause variation arises from identifiable, non-random factors—such as equipment malfunction, operator error, or material defects—that disrupt process stability and lead to deviations beyond expected limits.

To effectively monitor and control such processes, Shewhart control charts remain one of the most widely used tools in Statistical Process Control (SPC). These charts are particularly effective in detecting significant shifts in the process mean or dispersion, making them ideal for identifying special causes that require immediate corrective action (Montgomery, 2020). The design of the Shewhart chart, with its center line (CL), upper control limit (UCL), and lower control limit (LCL), enables practitioners to distinguish between natural process variation and abnormal deviations.

However, the performance of traditional Shewhart charts—based on classical estimators such as the sample mean and standard deviation is often compromised in the presence of outliers, non-normality, or measurement error, which are common in real-world industrial data. Under such conditions, conventional estimators become biased and inefficient, leading to inflated control limits, increased false alarm rates (Type I error), and delayed detection of true process shifts (Type II error).

To address these limitations, this study proposes a robust framework for Shewhart-type control charts by incorporating robust estimators of location and scale derived from auxiliary information. Specifically, the proposed estimators M_5 and M_6 utilize non-conventional measures of central tendency, such as the tri-mean, mid-range, and Hodges-Lehmann estimator, which are known for their resistance to extreme observations and high efficiency under contamination (Riaz et al., 2023; Khan et al., 2022).

A comprehensive Monte Carlo simulation study was conducted to evaluate the performance of the proposed estimators against traditional counterparts. The results consistently demonstrate that robust estimators outperform classical estimators in terms of key performance metrics, including:

- Average Run Length (ARL)
- Extra Quadratic Loss (EQL)
- Relative ARL (RARL)
- Performance Comparison Index (PCI)

The simulation outcomes reveal that the proposed robust estimators yield smaller values of ARL under out-of-control conditions, indicating faster detection of process shifts, while maintaining a stable in-control ARL, which minimizes false alarms. This confirms that a smaller value of performance measures—particularly ARL and EQL—is desirable for an efficient and **reliable** charting structure in process monitoring.

Furthermore, the robustness of the proposed estimators to outliers and measurement error **in** auxiliary variables enhances their applicability in practical settings where data quality is often imperfect. By leveraging auxiliary information—such as correlated process parameters—the proposed estimators achieve greater precision and stability, aligning with recent advances in auxiliary-based estimation under measurement error (Singh & Karan, 2021; Fuller, 2009).

FUTURE RESEARCH DIRECTIONS

While the current study focuses on detecting significant shifts using robust Shewhart charts, future research should explore the development of hybrid control charting schemes that simultaneously detect both large and small shifts. One promising direction is the integration of Shewhart and Non-Shewhart (e.g., CUSUM or EWMA) charts into a combined monitoring framework. Such a system would leverage the sensitivity of CUSUM/EWMA to small, gradual shifts and the immediate responsiveness of Shewhart to large deviations, thereby providing a more comprehensive and adaptive quality control solution.

Moreover, the use of auxiliary information should be extended beyond location parameters to the monitoring of dispersion (variance or standard deviation). Given that process variability is a critical determinant of quality, robust estimation of dispersion parameters—using auxiliary variables and robust scale measures like quartile deviation (QD) or inter quartile range (IQR)—can significantly enhance the effectiveness of control charts in detecting changes in process consistency.

Each industrial production process has two categories of process specificity, and the other is an irregular change due to a specific cause or production process. We use procedure of Shewhart control chart to monitor large shifts in the process. The simulation study of the estimators suggests that traditional estimators are less efficient than robust estimators i.e.,M_5 and M_6 . We conclude that to have a smaller value of performance measures for an efficient charting structure for process monitoring is desirable. The robust measures used in this research are strong against outliers present in the data. We recommended that this work might be prolonged to check large and small shifts in the system at the same time, by using combined form of Shewhart-Non Shewhart control charts. Moreover, the auxiliary information should be extended for monitoring of dispersion parameters.

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