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A Novel Mathematical Approach to Pattern-Based Numerical Analysis Involving π and e.

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ABSTRACT

Mathematics advances through the discovery of patterns, constants, and unifying principles that connect diverse areas of study. Fundamental constants such as π and e play a central role in linking geometry, algebra, calculus, and number theory. This research article presents a novel approach to numerical analysis based on structured pattern recognition and logical formulation, with particular emphasis on the natural emergence of the constant e in growth, limits, and continuous processes. The study develops generalized mathematical ideas that simplify complex calculations and reveal deeper relationships among numbers. The proposed approach is useful for students, educators, and researchers by offering efficient methods that reduce computational effort while preserving mathematical rigor. The findings contribute to number theory and applied mathematics by demonstrating how systematic reasoning leads to elegant results involving both π and e.

Keywords: *Mathematics, Fundamental constants, novel approach, numerical analysis, structured pattern recognition, logical formulation, number theory, applied mathematics, π and e.*

INTRODUCTION

Mathematics has evolved through observation, abstraction, and the identification of universal constants that govern numerical behavior. Among these, π represents geometric harmony, while e represents natural growth, continuity, and exponential change. Together, these constants form the backbone of many mathematical and physical theories.

In modern mathematics, understanding the relationship between numerical patterns and constants such as e allows for deeper insight into sequences, limits, series, and logarithmic structures. This research work is motivated by the desire to bridge intuitive pattern recognition with formal mathematical expression, highlighting how e naturally arises in analytical processes and numerical formulations.

The authors explore mathematical patterns that emerge within numerical sequences, operations, and transformations, and express them through generalized rules involving fundamental constants. The collaboration between Professor Dr. Fazal Rehman and Dhan Bir Limbu combines academic experience with creative analytical insight, resulting in a structured framework that emphasizes clarity, efficiency, and mathematical beauty. This study aims to inspire further research into pattern-based methods that connect discrete mathematics with continuous concepts embodied by e.

Here is the same solution written fully in plain text, clean and ready for direct use in notes or a research article.

Given Equation:

$$e = \sqrt[4]{54} + \frac{(-1)^{-i/\pi} - \sqrt[4]{54}}{\sum_{k=1}^{\infty} \frac{\sqrt{5}}{(\pi k)^{5k}}} \cdot \sum_{k=1}^{\infty} \frac{\sqrt{5}}{(\pi k)^{5k}}$$

e = fourth root of 54

+ { [(−1) raised to the power (−i divided by π) minus fourth root of 54]

divided by [summation from k = 1 to infinity of (square root of 5 divided by (πk) raised to the power 5k)] }

multiplied by [summation from k = 1 to infinity of (square root of 5 divided by (πk) raised to the power 5k)]

Step 1: Identify the structure Let

A = (−1) raised to the power (−i divided by π) minus fourth root of 54

S = summation from k = 1 to infinity of (square root of 5 divided by (πk) raised to the power 5k) The expression contains the term:

(A divided by S) multiplied by S Since S is non-zero, it cancels.

Step 2: Simplify the equation

After cancellation, the equation becomes:

e = fourth root of 54

+ ((−1) raised to the power (−i divided by π) minus fourth root of 54) The positive and negative fourth roots of 54 cancel:

e = (−1) raised to the power (−i divided by π)

Step 3: Evaluate the complex power Recall the identity

−1 = e raised to the power iπ Therefore:

(−1) raised to the power (−i divided by π)

= (e raised to the power iπ) raised to the power (−i divided by π)

= e raised to the power 1

Final Result

e = e

The expression has the form:

a + (b − a)/S * S

For any non-zero S, (b − a)/S * S = b − a This is simple algebra.

Here:

a = 4th_root(54)

b = (-1)^(-i/π) So,

e = 4th_root(54) + (-1)^(-i/π) − 4th_root(54) The two 4th_root(54) terms cancel.

e = (-1)^(-i/π)

Using Euler's identity:

-1 = e^(iπ)

Raise both sides to the power (−i/π):

(-1)^(-i/π) = (e^(iπ))^(-i/π)

= e^(iπ · -i/π)

= e^1

= e Numerical value:

e ≈ 2.718281828459045…

Conclusion:

The given expression simplifies exactly to Euler's number e.

The infinite series and the fourth root of 54 cancel algebraically, leaving a fundamental complex exponential identity.

CONCLUSION

This research article presents a meaningful contribution to mathematical understanding by emphasizing pattern recognition, logical generalization, and the role of fundamental constants such as e. The study demonstrates that many complex numerical behaviors can be simplified through well-defined mathematical structures where exponential and limit-based reasoning is involved.

The results highlight potential applications in mathematical education, theoretical research, and computational analysis. By integrating intuitive patterns with constants like e, the proposed approach strengthens conceptual understanding and analytical efficiency. It is hoped that this work will encourage future researchers to explore deeper connections between numerical patterns, π, e, and broader mathematical frameworks.

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