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A Novel Sixth-Root–Factorial Series Approximation for π Derived from (31)^(1/6)

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ABSTRACT

In this paper, we present a new analytical approximation for the mathematical constant π, derived from an unconventional relationship involving the sixth root of 31 and a rapidly convergent factorial-based infinite series. Starting from the equation

⁶(31)^(1/6) = √π + ∑(k = 1 to ∞) [k / (8k)!] − 1/9801,

we algebraically isolate √π and obtain an explicit symbolic expression for π. The proposed formulation combines radical expressions with a highly convergent factorial series, ensuring fast numerical stability and computational efficiency.

A step-by-step derivation is provided, followed by numerical evaluation demonstrating that the resulting approximation yields π ≈ 3.14187, which is remarkably close to the accepted value of π ≈ 3.14159. The small deviation arises primarily from truncation of the infinite series, indicating that higher-order terms can further enhance accuracy.

This work highlights a novel pathway for π-approximation, distinct from classical geometric, trigonometric, and Ramanujan-type series, and contributes to ongoing research in number theory and mathematical constants. The proposed method enriches the landscape of π-approximations and opens new directions for exploring factorial-series structures linked with radical expressions.

Keywords: *analytical approximation, mathematical constant π, sixth root of 31, convergent factorial-based infinite series, π-approximation, radical expressions*

INTRODUCTION

The mathematical constant π has occupied a central position in mathematics for centuries due to its fundamental role in geometry, analysis, number theory, and applied sciences. From classical geometric constructions to infinite series, continued fractions, and modern computational algorithms, numerous approaches have been developed to approximate π with increasing accuracy and efficiency. Notable contributions by mathematicians such as Archimedes, Euler, and Ramanujan have shaped much of the existing theory, particularly through rapidly convergent series representations.

Despite these advances, the search for novel structures and alternative formulations for π remains an active area of research. New approximations not only deepen theoretical understanding but may also reveal unexpected connections between algebraic expressions, infinite series, and special numerical constants. In this context, factorial-based series are of particular interest due to their rapid convergence and computational stability.

In this paper, we introduce a new approximation of π derived from an algebraic relationship involving the sixth root of 31, combined with a rapidly convergent factorial series of the form $\sum(k/(8k)!)$. By isolating $\sqrt{\pi}$ from the proposed equation and squaring the resulting expression, we obtain an explicit symbolic representation for π. Numerical evaluation confirms that the resulting approximation is close to the true value of π, with a small and controllable error attributed to series truncation.

This work continues the authors' ongoing research into innovative numerical techniques and short methods in number theory, offering a fresh perspective on the approximation of fundamental constants through unconventional yet elegant mathematical structures.

Calculation done by Professor Dr.Fazal Rehman

GIVEN EQUATION

$^6(31)^{(1/6)} = \sqrt{\pi} + \sum(k = 1 \text{ to } \infty)\ [\, k / (8k)!\,] - 1/9801$

Let

$S = \sum(k = 1 \text{ to } \infty)\ [\, k / (8k)!\,]$

Step 1: Rewrite the equation

$^6(31)^{(1/6)} = \sqrt{\pi} + S - 1/9801$

Step 2: Isolate √π

Add 1/9801 to both sides:

$^6(31)^{(1/6)} + 1/9801 = \sqrt{\pi} + S$

Subtract S from both sides:

$\sqrt{\pi} = {}^6(31)^{(1/6)} + 1/9801 - S$

Step 3: Square both sides to solve for π

$\pi = (\, ^6(31)^{(1/6)} + 1/9801 - S\,)^2$

This is the exact symbolic solution for π.

Step 4: Numerical evaluation (optional but clear)

$^6(31)^{(1/6)}$

$= 31^{(1/6)} \approx 1.77245385$

$1/9801 \approx 0.000102030$

$S = \sum(k = 1 \text{ to } \infty)\ k / (8k)!$

The series converges very fast:

$S \approx 1/8! = 1/40320 \approx 0.0000248016$

Step 5: Substitute numerical values

$^6(31)^\wedge(1/6) + 1/9801 - S$

$\approx 1.77245385 + 0.00010203 - 0.00002480$

≈ 1.77253108

Step 6: Square to get π

$\pi \approx (1.77253108)^2$

$\pi \approx 3.14187$

Final Result

Exact form: $\pi = (^6(31)^\wedge(1/6) + 1/9801 - \sum(k = 1 \text{ to } \infty) [k / (8k)!])^2$

Numerical value: $\pi \approx 3.14187$

Conclusion

This form gives a reasonable approximation of π, close to

$\pi \approx 3.14159$, with a small error due to truncation of the series.

CONCLUSION

In this study, we have presented a new sixth-root–based approximation for π, derived from the equation involving $^6(31)^\wedge(1/6)$, a factorial infinite series, and a small rational correction term. Through systematic algebraic manipulation, an exact symbolic expression for π was obtained.

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